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# Activity analysis models of educational institutions

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ACTIVITY ANALYSIS MODELS

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# I. INTRODUCTION

The economics of education is currently a rather popular topic. Most of the work that has been done in this area must be regarded as contributing to the macroeconomic aspects of education since it has concentrated largely on the relationship of education to the formation of human capital, on the returns to educational investments, or on the closely related matter of national educational decision making. This thesis is more concerned with the microeconomic aspects of education. It deals with decision making within individual educational institutions. The specific educational units to be considered are departments and colleges.

The impact of this particular study will necessarily be quite small, but it may be appropriate to consider the sorts of benefits which might result from increased attention to the microeconomics of education.

Increased emphasis on the microeconomic aspects of education could prove useful for national educational planning. Very few of the educational institutions in the United States are operated by the Federal government. As a result, federal agencies must often resort to rather indirect means when trying to achieve federal objectives. If reliable microeconomic models of educational institutions were available, they might be used to help estimate the effects of various possible federal instruments on the output and performance of educational institutions.

In contrast to the Federal government, most state governments are directly and explicitly concerned with the operation of universities and colleges. Increased efficiency of decision making in state universities and colleges may, over time, result in demonstrable improvement in their performance and presumably in better and more wisely directed support for higher education.

Some benefits could accrue to individual educational institutions. Economic models can hardly be expected to replace human decision makers. However, they may provide organized ways of looking at some of the decision problems, or at some aspects of some problems faced by educational institutions. Even if the sorts of problems which can best be handled by models turn out to be only the most routine ones, use of economic models may yield benefits by allowing more of the effort of educational decision makers to be directed toward the more difficult problems.

This study will present models which are intended to be of benefit primarily to individual educational institutions.

## II. GENERAL NATURE OF THE PROBLEM

The allocation problems of educational institutions are in some respects similar to the allocation problems faced by firms, households, and governments. In all cases the persons or institutions involved are faced with the problem of allocating scarce resources among alternative and competing ends or uses. Nevertheless, the task of formulating the allocation problems of educational institutions in such a way that they become amenable to economic analysis seems more difficult than that of formulating the allocation problems of firms, households, and governments. The fact that few persons have attempted it may at least suggest that this is the case.

There are, broadly speaking, two sorts of problems which may be encountered. On the one hand, there is the view that educational institutions should not be subjected to the same sort of analysis as the firm and the household. On the other hand, there is the (not unrelated) problem of finding suitable data and other information about educational institutions.

It is not clear just what aspect of education is supposed to render economic analysis inappropriate. Allan M. Cartter (11, p. 481) writes:

> "Some of our academic colleagues would deny the relevance of economic rationality to such a serious matter as education - economics is for the world of wheat, automation, and stock markets, they would argue, while higher education is the world of humane learning, scholarly inquiry and freedom of the spirit."

It is true that the outputs of educational processes and the processes themselves are less tangible than those found in manufacturing. But it is unlikely that this alone has been the source of all reluctance to study the economics of educational institutions. The fact that educational institutions are different should stimulate their study rather than discourage it.

It is possible that economists (and others) are reluctant to study educational institutions because they feel that the study of educational institutions would adversely affect the institutions themselves. Since many of the persons otherwise qualified to study educational institutions are employed by such institutions, the imputation of special pleading may be made if the findings are in some sense favorable to them. On the other hand, they may fear that only a part of the total value of the outputs of their institutions can be measured, and the less important part at that. If so, administrators might plan, and legislators appropriate, only for the vocational components of institutional programs.

However, it seems at least equally likely that increased knowledge of the quantifiable aspects of their institutions would increase faculty interest in, and influence upon, educational policies and procedures. Such facts as are compiled about college and university operations at present seem to be used primarily by registrars, business officers, deans, and higher administrators. As a rule they are not designed or disseminated in such a way as to facilitate faculty participation in resource allocation and educational policy except on an *ad hoc* or piecemeal basis.

Some members of the academic community may also fear that quantitative studies of their institutions would be used against them by legislators and taxpayers. Oversimplification or distortion of the findings might result in loss of revenue for the institutions, or in greater participation by elected officials in decisions previously left to the officers and faculty of the institutions. It may be well to note that neither of these eventualities can be presumed undesirable from the standpoint of individual voters unless they are unaware of the consequences of the actions. However, employees of educational institutions can hardly be expected to favor either of these outcomes. They are inclined to feel that too few resources are allocated to education already, and that educational institutions are already subject to too much control by elected officials.

Besides having to overcome the view that educational institutions should not be the basis for scientific investigations, anyone attempting to conduct such an investigation is also faced with a lack of data. Not only is there a paucity of collected data, but, in addition, that which has been collected was collected mostly for accounting and control purposes and thus is not always very useful for other purposes.

It might seem, therefore, that initial studies of educational institutions should be confined to data collection. This is not the only direction which can be taken, however, and is perhaps not even the best course to pursue. Some idea of the use to which the collected data might be put is likely to aid data collection. This suggests that some emphasis on model building prior to data collection might be appropriate.

There is another reason for proceeding with model construction before much effort is expended on data collection. It may not be sufficient to discover and record the values which various variables have had at various times and at various institutions. This sort of information is of course useful since it may provide some indication of the feasible values of these variables and permit checks on information received from other sources. It may not, however, be able to provide sufficient information about the range of feasible values. When this seems to be the case, other sources must be sought. Fortunately, such sources are available in the form of educators themselves. A good deal of the information needed is likely to be known in one form or another by faculty members and administrators of educational institutions. Some of this information is known to them simply because of their familiarity with their particular institutions. Other information is known to them because they are involved in determining the characteristics of the educational processes of their institutions and in formulating the goals and objectives of their institutions.

The process of obtaining information from educators is not without problems. The major problem is that educators seem to view educational institutions as consisting of one complex process whereas a model may need to view these institutions as being the framework in which many moderately interconnected processes are used.

The information gathering problem is analogous to that which would be faced by a person trying to gain information about the production function, preference function, quantities of commodities consumed, and prices of outputs and and inputs on the mythical island inhabited by Robinson Crusoe. Assuming that no shifts have occurred in either the production function or utility function, Crusoe could probably give accurate information, based either on his records or on his plans for the next period, about the equilibrium levels of all outputs and inputs. He probably could not as easily give accurate information about the equilibrium price ratios implied by the chosen levels of inputs and outputs, nor about his complete preference function or production function simply because he has little need to think about these things. He need not be concerned with price ratios since his island is a closed economy without exchange. However, if an attempt were made to construct an economic model of the island, somewhat more complete information about the production and preference functions would be needed.

The most expert source of information about these functions is Crusoe himself, but since he is not accustomed to thinking in terms of production and preference functions, the model builder would have to sort through any information received from Crusoe in order to determine whether it is (a) related to the production function, (b) related to the preference function, or (c) irrelevant. Educators are perhaps even less used to thinking about their activities in terms of production functions, products, prices, and related concepts. Thus it is likely that much of the information which could be received from them may be hard to identify as being related to one of the concepts which would be relevant to any model.

Because of the difficulty of obtaining reliable and relevant information either from previously collected data or records or from the "engineers" of the educational institutions (i.e. the faculty and administrators), it seems appropriate to first form some general idea of the set of concepts about which information is desired. While the specific information required will depend upon (a) the particular institution or type of institution studied, (b) the particular problem, process, or division of the institution which is studied, and (c) the mathematical or economic model used a basis for analysis and model construction, certain general types of information would seem to be useful in most cases. It would seem that information is needed about (a) the outputs of the institution, (b) the inputs used by the institution, and (c) the relationships which exist between the inputs and the outputs. The information needs in these areas are not entirely independent but separate treatment may allow a more understandable exposition of the needs in each area.

Consider first the outputs of the institution. Outputs of production processes are usually thought of as being tangible commodities whose qualitative and quantitative aspects are standardized and fairly well known. A somewhat wider view is relevant for the outputs of educational institutions. Some ways of looking at the outputs of educational institutions would direct attention to the relatively intangible aspects of their outputs. It is sometimes said that the outputs of educational institutions include the production, preservation, and transmission of knowledge. Not only are these outputs rather intangible, but, in addition, neither their qualitative nor their quantitative aspects are easily measured.

In order to construct an economic model of an educational institution, it is necessary to know what the outputs are and which are important for purposes of the model being constructed. Beyond that it is also necessary to know how to measure, or to decide upon some procedure for measuring, the outputs.

Closely related to the problem of specifying the outputs, is the problem of deciding what attitude should be taken toward them. Should some function of them be maximized? Should their output levels equal or exceed certain target values? Should some other attitude be taken with respect to the outputs? For the purposes of further exposition it will be assumed that the first attitude will be taken even though most previous models of non-market institutions have chosen the second attitude.[1] If the attitude is taken that some function of the outputs should be maximized, it is necessary to arrive at the function and at the parameters of that function.

In more direct terms, it can be said that arriving at a function to maximize is roughly equivalent to deciding what

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[1]The more common procedure is to minimize some function of the input while ensuring that the outputs satisfy certain targets. The more general approach would be to maximize some function of both outputs and inputs.

values or weights to associate with a unit of each type of output. Part of this problem is to decide on the source of the value of the outputs. If the approach is to be a normative one, the source of value for the purpose of the constructed model may be the market or a preconceived objective or social welfare function. If, on the other hand, the approach is to be a more positive one, the most immediate source of value is the objective function, or preference function, of the relevant decision maker. His objective function may in turn be influenced by the market or by some other source or system of value. Knowing the source of value of the outputs is not by itself sufficient to insure knowledge either of the function to be maximized or its parameters, although it can be a useful aid since it provides an indication of where to look for information about such a function.

There is also a need to specify the inputs, decide how to measure them, and arrive at their value. These problems are not, however, as acute as the analogous problems on the output side. This is largely due to the fact that present budgeting and accounting procedures have resulted in the categorization, measurement, and rewarding of some of the inputs. While the present treatment of the inputs may not always be exactly what is needed for any particular model, it provides, nevertheless, a good deal of information about the inputs of educational institutions.

Some inputs are not treated very explicitly in present budgeting procedures but may nevertheless be scarce enough to limit the output of the institution. Others may not currently be limiting the output of the institution but may become limiting if the output mix is changed or if the available amounts of other inputs are changed.[1] Ways must be found to measure and, whenever necessary, determine the value of these inputs.[2]

Information is also needed about the relationships which exist between the inputs and outputs. Which outputs are desired, how should they be measured, and what values do they have are questions which may be answered in part by persons or forces outside of educational institutions. Likewise, exogenous forces may specify the inputs to be used, the available quantities of these inputs, and the values of the inputs. Partly for historical reasons, but presumably primarily because of their greater competence in such matters, educators themselves have been responsible for

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[1]See (42) for a better treatment of this idea.

[2]The section which dealt with the outputs took, at least implicitly, the view that only the quantities of outputs were arguments of the preference function. It is likely that the objective or preference function may also include as arguments the quantities of some of the inputs.

the design of the educational processes which link the inputs and outputs of educational institutions.

It is not to be expected that the average educator can provide both the functional form and the parameters of the production functions. About the most that one can expect to obtain from educators, or from educational institution data, is some idea about some of the input-output combinations which are feasible.

Some of the relationships between inputs and outputs may not be technological relationships. Decision makers outside the particular part of the educational institution being considered may impose restrictions on the input-output combinations which may be used. They may, for example, place restrictions on the output mix or on the input mix. They may prohibit the use of otherwise feasible processes. Alternatively, the terms under which certain of the inputs are obtained may limit the uses to which these resources may be put.[1]

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[1]See (26) for some idea about how these restrictions might apply to research funds.

## III. REVIEW OF LITERATURE

The literature contains only a limited number of reports of attempts to construct economic or decision models of educational institutions or systems. On the other hand, there is a large and ever increasing volume of literature dealing with other aspects of the "economics of education". It appears that (with a few exceptions) most of the publications on this broader topic contribute little information and few concepts useful in the present study. Nevertheless, here and there, there are to be found a few grains of information and a few concepts which may prove useful.

This section consists of two parts. The first part is devoted to those contributions which have provided information or concepts useful to the present study. The second part will review those publications which seem to have the most to contribute to this study. No attempt will be made to provide a complete review of, nor to criticize thoroughly, the studies mentioned. Instead the emphasis will be placed on those portions of the studies which have provided, or which have attempted to provide, information or concepts which might be useful for economic models of educational institutions.

Few (if any) comprehensive lists of the outputs of educational institutions have ever been published. The literature does contain several different views about what the outputs are. Kidd (27, pp. 25-26) suggests that the functions of American universities are preservation of knowledge, transmission of knowledge, production of knowledge, and service to the community. Schultz (35, pp. 39-42) lists the functions of the educational establishment as research, discovery and cultivation of potential talent, increasing the capability of people to adjust to changes in job opportunities associated with economic growth, recruiting and instructing students for the teaching profession, and furnishing high level manpower for a country. Williams (42, p. 186) suggests that the outputs of educational institutions include (a) first degree graduates with pass degrees or honors degrees of class one, two, or three, (b) higher degree graduates in different subjects with masters degrees, doctorates, or diplomas for advanced studies, (c) research and writings of staff members.

The main difference between these lists of outputs or functions is the degree of finality of the outputs on the lists. Transmission of knowledge surely involves both written instruction (publications) and oral instruction (classroom teaching) by faculty members. Presumably at least the oral instruction is instrumental in producing some of the effects listed by Schultz. Students may be regarded as carriers, vectors, or agents through which teaching is able to produce high level manpower, new teachers, flexibility for economic growth, and other capabilities. Currently, the most common doses of, or exposures to, teaching received by students are the amounts required by various degree programs. Degrees may thus be regarded as outputs intermediate between teaching and the outputs listed by Schultz.

If the various sorts of outputs could be produced only in some fixed ratios to each other, it would not matter which view of the outputs is adopted. This is not the case, however, and the result is that even when one view is adopted, the others cannot be completely ignored. One cannot consider degrees awarded to be a homogeneous sort of output since it is possible that even within the requirements set by any particular degree program the amount of high level manpower per degree, the amount of flexibility per degree, and the amounts of other *desiderata* per degree may not be fixed but may vary somewhat. To the extent that the ratios of these latter outputs can be influenced, it may be appropriate to classify degree outputs not only by level and subject but also by the amounts of these other outputs per degree.

Most of the conceptual problems encountered in the measurement of outputs are problems of deciding what to measure, or of deciding what weights to give to each of several measurements. For example, for some purposes it would be convenient to have an aggregate measure of the human capital which can be attributed to education. Bowman (9) suggests the following alternative measures: (a) number of school years completed, (b) number of efficiency equivalence units, (c) base year lifetime incomes, (d) base year production costs, and (e) current real production costs. Alternative (a) suggests measuring human capital by measuring one of the inputs (years of schooling) involved in its production. Alternatives (d) and (e) suggest that production costs be used as a measure. Alternatives (b) and (c) suggest using productivity or market value as a measure of human capital.

Likewise the conceptual problems involved in determining the value of educational outputs are problems of deciding what aspects of the outputs are valuable, and of deciding upon the source of this value. Several writers have argued that a significant portion of the benefits of education accrue not to the educated person, but to his neighborhood, to his community, or to society as a whole. Blaug (6) and Denison (16) have suggested that education contributes to economic growth. Folsom (18) has argued that "everyone should help pay for education since everyone benefits". Eckaus (17) claims that wage and salary differentials between educated and uneducated persons are not accurate indicators of the differences in marginal productivity between educated and uneducated persons.

Other persons, including Professor Friedman (20) have claimed that, at least at the college level, most of the benefits of education accrue to the person who receives the education.[1] They argue that the major limitation of the market's evaluation of education is due to the fact that loans for investments in education and in other forms of human capital cannot be obtained as easily, nor at the same interest rate, as loans for investments in physical capital. These persons do not deny the existence of third party effects due to education but instead claim that they are small and that they may, in some cases, not even be positive.[2]

Several persons have attempted to estimate the value of education.[3] Most of the investigators have been concerned with the estimation of increases in lifetime incomes due to obtaining an undergraduate degree. Some of the investigators have chosen rate of return calculations to summarize their results; other have chosen present value calculations.

Those using rate of return calculations have reported

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[1]See also (40) and (44).

[2]See (2, p. 103) for some of the negative effects which could accrue from education.

[3]See, for example, (3), (23), (24), (32), (41), and (45).

rates of return ranging from about 5% to 14.5%.[1] Two factors, other than differences in the samples used, have undoubtedly contributed to the differences in results. Generally, the higher rates of return have resulted from studies which have largely ignored the effects of income-affecting variables other than education. The 5% figure is due to Hunt (24). His analysis is different from most others. For undergraduate education he attempted to estimate the rate of return associated with increased expenditure per pupil.

Hunt calculated rates of return on graduate studies using the more conventional approach.[2] For Master's degrees he obtained rates of return of about zero to minus one percent; for Ph.D. degrees the calculated rates of return were about one to two percent.

Those using the present value approach reported present values (at age fourteen) for undergraduate education (for males) ranging from $1,700 to $100,000. The larger figure is attributed to Glick and Miller (21, p. 310). It was obtained from U.S. census data by using a zero

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[1]See (24 and (3), respectively.

[2]The rate of return as most commonly computed for a particular level of education tends to estimate the rate of return that would be forthcoming from additional investment in education if that investment allowed more persons to attain that level of education.

interest rate. Houthakker (23, p. 28) verified this estimate, and also demonstrated that applying an 8% interest rate to income after income tax would reduce the estimate to $3,000. Wilkinson (41, p. 562) used data for Canada and obtained estimates of $12,000 and $1,700 when using interest rates of 5% and 10%, respectively, for discounting income after income tax.

Several writers have critized the rate of return approach.[1] Some of the criticism has been directed at the data used for rate of return calculations. Others have insisted that the rate of return approach is less relevant than the present value approach. Part of this criticism is based on the fact that for net return series which have several sign reversals[2] the rate of return need not be unique. This criticism is not particularly important for the present work. It is generally felt that an individual's net return stream due to an additional increment of education has only one sign reversal. Furthermore, if multiple sign reversals occurred, the statistical techniques used could not be expected to verify their existence with any reasonable

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[1]See especially (22), (31), and (46).

[2]A net return series $R_t$ has at least one sign reversal unless $R_t>0$ for all relevant t or $R_t<0$ for all relevant t. It has more than one sign reversal, if for some $t_1<t_2<t_3$, $R_{t_1}$, $R_{t_3}>0$ and $R_{t_2}<0$ or if for some $t_4<t_5<t_6$, $R_{t_4}$, $R_{t_6}<0$ and $R_{t_5}>0$.

degree of certainty.

Neither of these approaches will be used directly in this study. For studies of this type, however, the present value approach seems more useful. The advantage of this approach is that it is more readily adapted to models in which some of the elements of the net returns stream are endogenously determined. If some discount rate $r^*$ is selected as being most appropriate, then the present value of an investment is a linear function of the form

$$PV(r^*,0) = \sum_{t=0}^{T} ( \gamma_t c_t + \rho_t R_t) \text{ where}$$

$$- \gamma_t = \rho_t = \frac{1}{(1 + r^*)^t}$$

The problems of deciding what the inputs are, and what their values are, have not received as much attention in the literature as the analogous problems on the output side. Some contributions have been made on the input side, however. Cartter (10) has recently completed a study of the quality of graduate faculties and graduate programs. His study not only indicated that differences in the quality of various graduate faculties had been perceived but also that the differences in ratings were largely independent of the age, rank, or geographical location of the raters.

Becker (3) and others have included the value of student time in their estimates of the costs of a college education.

In their studies, the value of student time accounts for a large part of the total costs of a college level education. Becker (4) has recently shown that, under certain assumptions, the value of the time used is an important component of the cost of consuming goods as well. Although his result seems to overstate the actual situation, there seems to be no doubt that the opportunity cost of the students' time should be important in his decision about college attendance.

Contribution by Stone and Blaug suggest that the problem of discovering the value of the student input is not as simple as Becker suggests. Blaug (6) warns that private motivations are important to any policy which seeks to change the educational level. Stone (38) suggests that the concept of the "demand for places"[1] is useful when considering the student inputs. Stone (38, p. 186) also writes:

> ". . . . the economics of demand (for education)[2] are rather shadowy since . . . families of most

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[1]The "demand for place" is the demand by students for admission to educational institutions.

[2]The words (for education) were not included by Stone but were added here to make the meaning somewhat clearer.

> students will bear directly only a small part of the costs of education. On the whole this seems to me a rather desirable state of affairs but it throws back on those responsible for general social and economic policy the decision of how much of the national resources should be devoted to education. This brings us to an area of social cost-benefit analysis in which market values are a necessary but by no means sufficient ingredient."

The number of articles and other publications dealing with production functions or other relationships between inputs and outputs in colleges and universities is rather small. Apparently most of that sort of research has concentrated at the elementary and secondary school level. The parameter most frequently mentioned in publications dealing with higher education seems to be the student/staff ratio and its variants. Bolt *et al.* (7) and Stoikov (37) have included a production function for scientists in the models which they have designed to aid in the allocation of scientific effort. The production function used was essentially: $P = r\ E$[1] where

P = the rate of production of new Ph.D.'s

E = the number of educational scientists (full time equivalent)

r = the number of new Ph.D's per man year of teaching

Maul (34) and Cartter (12) have used essentially the same

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[1] The Bolt *et al.* version is presented here.

sort of function in their analyses of the future quality of college faculties (measured by % of faculty with Ph.D.'s). Cartter's version of the production function is:

$F_t = F_{t-1} + f(E_t - E_{t-1})$ where

$F_t$ = the number of faculty members in year t

$E_t$ = the enrollment in year t

f = the staff/student ratio

A few persons have constructed more or less complete models of educational institutions or educational systems. Some of these will be reviewed briefly here.

Mrs. Adelman (1) has constructed educational planning models for Argentina. She treated the educational system as one sector in a linear programming formulation of the whole economy. The major difference between each of her models is the particular objective function chosen to be maximized.[1]

Bowles (8) has constructed an educational planning model for Northern Nigeria. The value of the objective function of the model equals the net contribution of the educational sector to the present value of present and future national income. The activities in the model are

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[1]Several different objective functions were used. One of them was designed to maximize the rate of growth; another was designed to maximize the employment rate.

limited to the processes of the educational sector. The constraints include **restrictions** on the supply of internally produced inputs such as teachers and students as well as restrictions on the supply of exogeneously produced inputs such as funds to support the schools and on the supply of beginning (6 year old) students. The source of value of the outputs is the productive sector of the economy.

Intrilligator and Smith (25) have constructed a model which is designed to help guide the allocation of scientific effort between teaching and research. Their model emphasizes the social benefits as the source of value of educational outputs. They assume that the output from teaching and research can be measured by the number of (full time equivalent) teaching and research scientists, respectively. Their model is:

$$\max_{o<b_0\leq b\leq b_1<1} W = \max_{o<b_0\leq b\leq b_1<1} \left\{ F\left[R(T), E(T)\right] + \int_0^T I\left[R(t), E(t), t\right] dt \right\}$$

subject to

$$\dot{E}(t) = bf\left[E(t), t\right] - dE(t)$$

$$\dot{R}(t) = (1-b)f\left[E(t), t\right] - dR(t)$$

where $\dot{E}(t) = \dfrac{\delta E(t)}{\delta t}$ , $\dot{R}(t) = \dfrac{\delta R(t)}{\delta t}$

$b = b(t)$ is the path over time of the percentage of new scientists allocated to teaching

$f = gE(t)$ is the educational production function

$d =$ the rate of exit from the profession

$W =$ welfare

$t$ = time

$t = o$ initial (present) time

$t = T$ terminal (future) time

$R(T)$ = research scientists at time $t$

$E(t)$ = teaching scientists at time $t$

$F$ = is a future component of welfare

$I$ = is the rate of increment to welfare from time $t = 0$ to $t = T$

Using the techniques of control theory, optimal time paths for b were derived under several different specifications of W.

Stone (38) has constructed a dynamic input-output model of the British educational system. The numbers of persons leaving the educational system after receiving various amounts of education can be considered the final products of the system,[1] but the demand for places seems to be a more direct determinant of amounts of the various non-student education inputs which are required. For the purposes of the model the demand for places is treated partly as a set of epidemic processes and is assumed partly determined by economic prospects. The demand for places at any given level is treated as an epidemic process of the

[1]Actually $h^{-1}g$ fits better the formal requirements of a final product vector since the flow equation of the model is $S = h^{-1}JES + h^{-1}g$ where $S$ = stock of students at the beginning of a year, $h$ = a matrix of survival rates, $J = \begin{bmatrix} 0 & 1 & & 0 \\ & \ddots & \ddots & 1 \\ 0 & & & 0 \end{bmatrix}$ $g$ = graduate leavers at the end of the year, and $ES_t = S_{t+1}$.

form:

$$\Delta P_i = a_i \, P_i (1 - b_i - P_i) \text{ where}$$

$P_i$ = the proportion of students who attended the ith process during the last year

$(1 - b_i - P_i)$ = the proportion who did not attend but who have the ability and who in happier circumstances would wish to attend

The demand for places within any level of education is yielded by the relation:

$$E \, r_j = \alpha_j \, r_j + (1 - \alpha_j) \, r_j^* \text{ where}$$

$$E \, r_{j,t} = r_{j,t+1}$$

$r_j$ = the proportion of students who demand places in the jth specialty

$r_j^*$ = the equilibrium proportion given present prospects, that is the proportion which would exist on an objective assessment of prospects,

$\alpha_j$ = a parameter reflecting the rate at which actual proportions adjust to equilibrium proportions.

The demands $e_d$ and supplies $e_s$ of different types of teachers can be derived from the relations:

$$Ee_d = AEs \text{ and}$$

$$Ee_s = (I - C)e_s + Bg \text{ where}$$

A is a matrix of staff-to-student ratios

C is a matrix of wastage rates

B is a matrix reflecting the proportions of the graduates at various levels who enter the various types of teaching

g is a vector of graduates

E is an operator which advances the time subscript of any variable which preceeds by one year

Winkelmann (43) has constructed a model designed to aid in the allocation of the time of faculty members among various available teaching and research assignments. Whereas the models discussed above are concerned with the allocation of educational resources on a large (usually national) scale, his model deals with allocations decisions faced by an individual department chairman. Specifically, the model is designed to allocate fixed faculty resources among research and teaching assignments. It is assumed that the department under consideration has not only a fixed set of faculty resources, but has, as well, a fixed set of teaching requirements to fulfill. It is also convenient to assume that the set of research opportunities is fixed.

The inputs considered in the model are the individual faculty members. The unit of measurement of the inputs is the amount of time required to teach one section of any course, or to complete any single research assignment. The outputs are teaching and research. They are measured in two ways. One method measures outputs by the amount of faculty resources required to perform them. Under this method of measurement there is little scope for optimization. Both the quantities of inputs available and the quantities of inputs required are fixed, thus all that can be done is to determine whether or not a feasible solution exists.

The second method uses value as the measure of the outputs. Under this method total output is measured in terms of its contribution to the present value of future national income. The model thus uses this second measure of output as its objective function while the first method of measurement is instrumental in defining the constraints of the model.

In order to implement this model it is necessary to be able to estimate the value of each feasible allocation. Its author presents a procedure by which a linear objective function based partly on market information and partly on the department chairman's subjective evaluations could be derived.

Plessner and Fox (33) have constructed a linear programming model of a department. Their model deals with output level decisions. The outputs included in their model were degrees awarded and hours of research completed. The degrees awarded were classified by degree level and, in the case of Ph.D. degrees, by means of support. Thus the degree outputs included B.S. degrees, M.S. degrees, Ph.D. degrees awarded to persons not on appointment (i.e. candidates supported by fellowships, other employment, or personal resources), Ph.D. degrees awarded to persons who had been instructors, and Ph.D. degrees awarded to persons who had held appointments as research assistants.

The model's inputs included office space, students, and faculty time. It was assumed that the department was endowed with certain quantities of these inputs. It was further assumed that additional amounts of all resources except students could be "purchased" at prevailing market prices.

The number of hours of research completed was required to equal or exceed $\alpha_1$ times the number of hours of faculty time with which the department was originally endowed plus $\alpha_2$ times the number of "purchased" faculty hours.[1] The other outputs and all inputs except students were regarded as flexible targets. The levels of these outputs and inputs were to be selected so as to maximize (subject to the constraints in the model) the department's net contribution to the present value of national product.

One version of the model dealt with a four year planning period. Output decisions made prior to the first year of the planning period provided constraints in addition to those provided by resource limitations.

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[1]The coefficients $\alpha_1$ and $\alpha_2$ were set equal to minimum proportions of their time that "original" and "new" faculty members, respectively, would insist on devoting to research.

## IV. MATHEMATICAL TOOLS TO BE USED

One possible use of a model of an educational institution would be to aid in the selection of values of certain variables (such as output levels, and the amounts of faculty time and other resources to be allocated to various functions) from among those values which are feasible, so as to maximize the value of some preference function. Formally, this involves finding a vector $x^*$ such that $f(x^*) = \max_{x \in X} f(x)$ where $f(x)$ is the institutions' preference function and $X$ is the set of feasible values of $x$.

Economists are rather familiar with this sort of problem. Much of economic theory is based on this sort of mathematical model. In the case of consumption theory $f(x)$ is a function representing the consumer's preference ordering and $x$ is the set of commodity bundles which the consumer can afford. In the theory of the firm $f(x)$ is often the profit function and $X$ is the set of feasible input-output combinations.

Often economists are not so much concerned with finding the maximizer $x^*$ as with examining the characteristics of the solution and the symptoms of optimizing behavior. Thus economists have long used calculus and the theory of constrained maximization to derive the "marginal conditions" and to determine the form and characteristics of demand or supply functions. More recently some elements of set theory have partially replaced calculus as a tool for examining the characteristics of optimum solutions.

Activity analysis and convex programming have also been used to examine the characteristics of optimum solutions. Some formulations of these two tools have also found extensive use in applied economic research. In this latter application linear programming and other formulations of activity analysis and/or convex programming have been used to calculate the maximizer $x^*$.

Activity analysis as formulated by Koopmans has found some use in economic theory.[1] Some of the features and results of his formulation are useful in the present study.

Koopman's activity analysis model has the form:

$$\max f(y_{fin})$$

$$\text{subject to } Ax = \begin{bmatrix} y_{fin} \\ y_{int} \\ y_{pri} \end{bmatrix}$$

$$y_{int},\ y_{fin},\ x \geqq 0,\ y_{pri} \geqq \eta_{pri} < 0$$

where f can be any function such that

$$f(\bar{y}_{fin}) > f(\hat{y}_{fin}) \text{ if } \bar{y} > \hat{y}.^2$$

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[1]This section is based on Koopmans (28).

[2]This requirement that the objective function not exhibit saturation of demand for any of the final commodities is not too important to the present study and will often be relaxed.

Koopmans uses two concepts, the commodity and the activity, to provide a bridge between the linear model $y = Ax$ and observable production processes. Each commodity is assumed to be homogeneous qualitatively and continuously divisible quantitatively. Commodities include primary factors, intermediate products, and final products. Primary factors are commodities which can be made to flow into the economy from nature or from the outside world. $\eta_{pri}$ is a vector of the rates at which the primary factors flow into the economy. $y_{pri}$ is the vector of the rates of use of primary factors. Final products are those desired commodities which are not primary factors. $y_{fin}$ is a vector of the rates of production of final products. Those commodities which are neither primary factors nor final products are called intermediate products. Each intermediate product is an output of at least one activity and an input of at least one other activity. $y_{int}$ is a vector of net rates of production (rates of production minus rates of use) of intermediate products.

In Koopmans' treatment of activity analysis an activity consists of the combination of certain commodities (inputs) in fixed ratios to produce certain other commodities (outputs) in fixed quantitative ratios to the inputs.[1]

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[1]See (28, p. 47).

Thus an activity is an elementary production function which is homogeneous of degree one. By admitting a large number of activities any production function which exhibits non increasing returns and is homogeneous of degree one can be approximated as closely as desired in an activity analysis model.

Koopmans uses several other definitions in his discussion of activity analysis. Some of these are presented below:

Def. 1. "A point y in the commodity space is called possible in a technology A if there exists a point x in the activity space satisfying

$$y = Ax \ , x \geqq 0"[1]$$

Def. 2. "A point y in the commodity space is called attainable if there exists a point x in the activity space such that

$$y = \begin{bmatrix} y_{fin} \\ y_{int} \\ y_{pri} \end{bmatrix} = Ax, x \geqq 0, \ y_{fin} \geqq 0$$

and such that $y_{int}$ and $y_{pri}$ satisfy (5.1) and (5.2) respectively"[2]

Def. 3. "A point y in the commodity space is called efficient if it is attainable and if there exists no attainable point $\bar{y}$ such that

$$\bar{y}_{fin} - y_{fin} \geq 0."[3]$$

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[1]See (28, p. 47).

[2]See (28, p. 79). (5.1) and (5.2) require that $y_{int} = 0$ and $y_{pri} \geqq \eta_{pri} < 0$.

[3]See (28, p. 79).

Some of the results of the study of activity analysis may also be useful. It has been shown[1] that an attainable point y is efficient if and only if there exists a vector P such that

$$P'A \leqq 0,\ P'y = 0,\ P_{fin} > 0,\ P_{pri=} \geqq 0,\ P_{pri>} = 0$$

where $P_{fin}$, $P_{pri=}$, $P_{pri>}$ are respectively those components of the vector P which correspond to find commodities, exhausted primary factors, and unexhausted primary factors. It has also been shown[2] that an attainable point y is efficient if and only if there exists a positive vector $\pi_{fin}$ such that $\pi'_{fin}y_{fin}$ reaches a maximum, within the attainable point set, in y.

A version of activity analysis widely used in applied economic research is linear programming. Its wide use has undoubtedly both stimulated and resulted from the development of operational solution procedures and computer programs. Linear programming is a very versatile tool since it can be applied not only to linear problems but can also provide approximate solutions to some non-linear problems. By virtue of one of the results noted above it is apparent that linear programming can be used to find the efficient points

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[1]See (28, p. 62).

[2]See (28, p. 86).

in more general activity analysis problems.[1]

The general form of a linear program is

(4.2) $$\max_{x \in X} c'x$$

$$X = \left\{ x \mid Ax \leqq b,\ x \geqq 0 \right\}.$$

The dual of (4.2) is:

(4.3) $$\min_{v \in V} v'b$$

$$V = \left\{ v \mid A'v \geqq c,\ v \geqq 0 \right\}.$$

Several theorems have been established which deal with linear programs. The ones which seem the most useful for this study are those which deal with the relationships that can exist between the solution of the primal (4.2 for example) and the dual (4.3 for example) of a linear program. Some of the possibilities are summarized in Table 1.

Dantzig and Wolfe (15) have developed an algorithm for solving certain types of large linear programs. This algorithm is most useful for those linear programs which

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[1]Charnes and Cooper (13) have developed a procedure for finding these efficient points.

Table 1. Relationships which can exist between the solution to primal (4.2) and dual (4.3) forms of linear programs.[a]

| | 4.2 has a feasible solution ($X \neq \emptyset$) | 4.2 has no feasible solution ($X = \emptyset$) |
|---|---|---|
| 4.3 has a feasible solution ($V \neq \emptyset$) | $\max_{x \in X} C'x = \min_{v \in V} v'b$ | $\min_{v \in V} v'b \to -\infty$ |
| 4.3 has no feasible solution ($V = \emptyset$) | $\max_{x \in X} C'x \to +\infty$ | possible |

[a]Adapted from (14, p. 128).

involve an A matrix of the form

$$A = \begin{bmatrix} \bar{A}_1 & \bar{A}_2 & \cdots & \bar{A}_n \\ \underline{A}_1 & & & \\ & \underline{A}_2 & & \\ & & \ddots & \\ & & & \underline{A}_n \end{bmatrix}$$

and for which the number of rows in the $\bar{A}_i$'s is small relative to the total number of rows in A. Their algorithm takes advantage of the fact that, if $\underline{X}_i$ is bounded, then for any vector $x_i$ belonging to $X_i$ there exist scalars $\lambda_{i1}, \ldots$ $\lambda_{im_i}$ such that

$$x_i = \sum_{j=1}^{m_i} \underline{x}_{ij} \lambda_{ij}$$

$$\sum_{j=1}^{m_i} \lambda_{ij} = 1, \; i = 1, 2 \ldots n$$

$$\lambda_{ij} \geqq 0 \quad j = 1, 2 \ldots m_i, \; i = 1, 2 \ldots n$$

where the $\underline{x}_{ij}$'s are the $m_i$ extreme points of the set $\underline{X}_i$. For a linear program suitable for the application of their algorithm the sets $\underline{X}_i$ are given by

$$\underline{X}_i = \left\{ x_i \;\middle|\; \underline{A}_i x_i \leqq \underline{b}_i, \; x_i \geqq 0 \right\}$$

where $x_i$ and $\underline{b}_i$ are the relevant portions of the x and b vectors of 4.2.

Dantzig and Wolfe have shown that if the sets $\underline{X}_i$ are bounded for $i = 1, 2 \ldots n$, then problem 4.2 is equivalent to 4.5.

$$(4.5) \quad \max \sum_{i=1}^{n} \sum_{j=1}^{m_i} \gamma_{ij} \lambda_{ij}$$

subject to

$$\sum_{i=1}^{n} \sum_{j=1}^{m_i} B_{ij} \lambda_{ij} \leqq \bar{b}$$

$$\sum_{i=1}^{m_i} \lambda_{ij} = 1, \; i = 1, 2 \ldots n$$

$$\lambda_{ij} \geqq 0, \; j = 1, \ldots m_i$$
$$i = 1, \ldots n$$

The $B_{ij}$'s and $\gamma_{ij}$'s are obtained by letting

$$B_{ij} = \bar{A}_i \underline{x}_{ij} \text{ and}$$
$$\gamma_{ij} = c_i' \underline{x}_{ij}$$

The major advantage of the algorithm is due to the fact that for many problems it is not necessary to use all $(\sum_{i=1}^{n} m_i)$ columns of 4.5 to obtain a solution to 4.5. An iterative procedure has been developed which involves, initially, the construction of only as many columns as are needed to obtain a feasible solution to problem 4.5.[1] Additional columns are then generated, as needed, during each phase of the procedure.

The Nth phase of the procedure has two parts, the first of which involves solving the dual of the Nth phase version of problem 4.7.

$$(4.7) \quad \max \sum_{i=1}^{n} \sum_{j=1}^{m_i\langle N\rangle} \gamma_{ij} \lambda_{ij}$$

subject to

$$\sum_{i=1}^{n} \sum_{j=1}^{m_i\langle N\rangle} B_{ij} \lambda_{ij} \leq \bar{b}$$

$$\sum_{j=1}^{m_i\langle N\rangle} \lambda_{ij} = 1, \; i = 1, 2 \ldots n$$

$$\lambda_{ij} \geq 0 \,, \; j = 1, 2 \ldots m_i\langle N\rangle^{2}$$
$$i = 1, 2, \ldots n$$

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[1]Their algorithm can also be used to obtain a feasible solution.

[2]$m_i\langle N\rangle$ equals the number of extreme points of the set $X_i$ which have been used to construct the Nth phase version of 4.7.

The second part of the Nth phase involves solving n subprograms. The Nth phase version of the ith subprogram is given by 4.8.

$$(4.8)\ \hat{\Omega}_i\langle N\rangle = \max_{x\in \underline{X}} \left[ c_i' - (\overline{V}\langle N\rangle)'\overline{A}_i \right] x_i - q_i\langle N\rangle$$

$\overline{V}\langle N\rangle$ and $q_i\langle N\rangle$ are elements of the vector, $V\langle N\rangle$, which solves the dual to the Nth phase version of 4.7. $\overline{V}\langle N\rangle$ is the element of $V\langle N\rangle$ which corresponds to the restriction $\sum\sum B_{ij}\ \lambda_{ij} \leq \overline{b}$, and $q_i\langle N\rangle$ is the element of $V\langle N\rangle$ which corresponds to the restriction $\sum \lambda_{ij} = 1$.

Those subprogram (extreme point) solution vectors $x_i\langle N\rangle$ which correspond to positive $\hat{\Omega}_i\langle N\rangle$'s are used to generate columns which are added to the Nth phase version of 4.7 to form the (N+1)th phase version of 4.7.

This iterative procedure continues until at the end of some phase, say the Mth, $\hat{\Omega}_i\langle M\rangle = 0$ for $i = 1, 2, \ldots .n$. The solution to the Mth phase version of 4.7 can be used to obtain a solution to 4.2. The solution vector to 4.2 is obtained by letting

$$x_i^* = \sum_{j=1}^{m_i\langle M\rangle} \underline{x}_{ij}\ \lambda_{ij}^* \quad \text{for } i = 1, 2 \ldots .n.$$

The procedure outlined above often must be modified when it is applied to problems for which not all of the sets $\underline{X}_i$ are bounded. If the set $\underline{X}_h$ is not bounded it is not possible to express every vector $x_h$ belonging to $\underline{X}_h$ as a (non negative weights) weighted average of the extreme points of $\underline{X}_h$. However, there always exist scalars $\lambda_{h1}$, $\lambda_{h2} \ldots \lambda_{hm_h}$, $\mu_{h1} \ldots \mu_{hk_h}$ such that

$$x_h = \sum_{j=1}^{m_h} \lambda_{hj} \; \underline{x}_{hj} + \sum_{l=1}^{k_h} \mu_{hl} \; \underline{x}_{hl}$$

$$\sum_{j=1}^{m_h} \lambda_{hj} = 1, \; h = 1, \ldots n$$

$$\lambda_{hj}, \mu_{hl} \geqq 0, \; \begin{array}{l} j = 1, 2 \ldots m_h \\ l = 1, 2 \ldots k_h, \end{array} \quad h = 1, 2 \ldots n$$

where the $\underline{x}_{hj}$'s are the $m_h$ extreme points of the set $\underline{X}_h$ and the $\underline{x}_{hl}$'s are the $k_h$ extreme points of the set $\underline{X}_h(0)$ $= \{ x_h \mid A_h x_h \leqq 0, 1' x_h = 1, x_h \geqq 0 \}$.[1]

Thus when not all of the sets $\underline{X}_i$ are bounded 4.2 is not equivalent to 4.5 but it is equivalent to 4.9.[2]

$$(4.9) \; \max \sum_{i=1}^{n} \left( \sum_{j=1}^{m_i} \gamma_{ij} \lambda_{ij} + \sum_{l=1}^{k_i} \gamma_{il} \mu_{il} \right)$$

subject to

$$\sum_{i=1}^{n} \left( \sum_{j=1}^{m_i} B_{ij} \lambda_{ij} + \sum_{l=1}^{k_i} B_{il} \lambda_{il} \right) \leqq \bar{b}$$

$$\sum_{j=1}^{m_i} \lambda_{ij} = 1, \; i = 1, 2 \ldots n$$

$$\left.\begin{array}{l} \lambda_{ij} \geqq 0, \; j = 1, 2 \ldots m_i \\ \mu_{il} \geqq 0, \; l = 1, 2 \ldots k_i \end{array}\right\} \; i = 1, 2 \ldots n$$

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[1] 1 is a column vector of ones.

[2] This may happen even when X is bounded.

The Nth phase of the modified procedure also has two parts. The first part involves solving the dual of the Nth phase version of problem 4.10.

$$(4.10) \quad \max \sum_{i=1}^{n} \left( \sum_{j=1}^{m_i\langle N\rangle} \gamma_{ij} \lambda_{ij} + \sum_{l=1}^{k_i\langle N\rangle} \gamma_{il} \mu_{il} \right)$$

$$\text{subject to } \sum_{i=1}^{n} \left( \sum_{j=1}^{m_i\langle N\rangle} B_{ij} \lambda_{ij} + \sum_{l=1}^{k_i\langle N\rangle} B_{il} \mu_{il} \right) \leqq \bar{b}$$

$$\sum_{j=1}^{m_i\langle N\rangle} \lambda_{ij} = 1, \; i = 1, 2 \ldots n$$

$$\left.\begin{array}{l} \lambda_{ij} \geqq 0, j = 1, 2, 3 \ldots m_i\langle N\rangle \\ \mu_{il} \geqq 0 \; l = 1, 2 \ldots k_i\langle N\rangle^{1} \end{array}\right\} i = 1, 2 \ldots n$$

The second part of the Nth phase of the modified procedure involves solving n subprograms. Whenever possible the subprograms will have the form of 4.8. For those cases (i.e. those values of N and i) for which unbounded solutions result to subprograms having the form of 4.8, subprograms having the form 4.11 are solved.

$$(4.11) \quad \max_{x_i \in \underline{X}_i(0)} \left[ c_i' - (\bar{V}\langle N\rangle)' \bar{A}_i \right] x_i$$

Extreme point solutions resulting from solving

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[1] $k_i\langle N\rangle$ equals the number of extreme points of the set $\underline{X}_i(0)$ which have been used to construct the Nth phase version of 4.10.

subprograms having the form of 4.8 are treated as described in the original procedure. Extreme point solutions which result from the solution of subprograms having the form of 4.11 are used to form columns which will be added (along with those resulting from solution of subprograms having the form 4.8) to the Nth phase version of 4.10 to form the (N + 1)th phase version of 4.10. The only difference between the two types of columns is that those resulting from extreme points of $\underline{X}_i(0)$ do not have a one (1) in that row corresponding to the restriction $\sum_{j=1}^{m_i\langle N\rangle} \lambda_{ij} = 1$ whereas those which result from extreme points of $\underline{X}_i$ do have a one in that row.

The termination conditions and the translation of solutions of 4.10 into solutions of 4.2 are exactly the same as for the unmodified procedure.

The last tool to be discussed in this section is the Tinbergen-type approach.[1] While this approach is not itself a mathematical tool it has proven useful since it allows the formulation of certain problems in such a way that they can be solved by mathematical tools. Its primary area of application has been in economic policy but it is easily applied to many other problems. The Tinbergen approach

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[1]This section is based on Fox, Sengupta, and Thorbecke (19, pp. 448-450).

consists of three basic elements.

The first element is the decision maker's welfare or preference function. The decision maker's welfare function is used since in many applications it is not reasonable to believe that the relevant social welfare function based on individual preferences can be found.

The second element is the classification of the variables. The first class consists of the exogenous variables. Included in this class are the policy means and other data. The policy means are those variables over which the decision maker has some control and by which he seeks to influence the target variables. The other data are those variables important to the problem but over which the policy maker has no control. The second class consists of the endogenous variables. Included in this class are the target variables and the irrelevant variables. The target variables are those which the decision maker wishes to influence. The irrelevant variables are not instruments, other data, nor targets but nevertheless required to complete the structural model.

The third element is the structural model, an economic model which defines the relationship which exist among all the variables of the system which the decision maker seeks to guide or influence.

## V. ADAPTING THE TOOLS TO THE PROBLEM

The fact that only a few tools were discussed in the preceding section does not mean that the other tools used by economists have nothing to contribute to the study of educational institutions. Instead it simply reflects an intent to rely primarily on activity analysis, and particularly on linear programming, in the present study.

Activity analysis has certain advantages which suggest that it might be useful here. Several of the variants of activity analysis, especially linear programming and to a lesser extent quadratic programming, are readily solvable by digital computers. Thus any problem formulated as a linear program can be rather easily solved. Moreover, the linear structural model common to all activity analysis variants is capable of approximating many relationships that are not linear. These considerations alone provide some incentive for the use of activity analysis.

The linear model common to activity analysis and some other tools provides a format by which many relationships existing in educational institutions can be fairly easily represented. It is possible in many cases to treat educational outputs as being the result of several inter-related processes each of which can be separately represented in an activity analysis model. As a result many of the coefficients in such a model may be simply related to commonly observed numbers. This may allow easier explanation of the resulting model to educators and other interested persons. As a result educators may be able to provide constructive criticisms of educational models.

Activity analysis is not without limitations. Perhaps the most serious limitation of activity analysis for educational models results from its inherent assumption of infinite divisibility of commodities and activities. Some of the outputs of educational institutions are not divisible and it is not sensible to believe that all of the processes of educational institutions are infinitely divisible. However, the assumption of divisibility may not cause much loss of realism in some cases. In those cases where noninteger solutions may be unacceptable it is possible to limit the solutions to integer solutions by using integer programming procedures.

While activity analysis can be used in the formulation and solution of models of educational institutions, the elements of an activity analysis model of an educational institution need not correspond exactly to the elements of any problem faced by the institution. In order to make this point a bit more clearly, one approach to the translation of such a problem into an activity model will be presented below.

The "commodities" (to use Koopmans' term) in such an activity analysis model should include all of the inputs, outputs, and intermediate products relevant to the problem. "Commodities" may include teaching, counseling, and other services which are somewhat intangible. In fact most of the commodities relevant to educational models are likely to be of this type. Any one commodity should, however, be fairly uniform in whatever qualitative aspects are considered important. This can be accomplished by treating a non-uniform product as several different commodities.

It is desirable that the objective function of a model be such that a solution to the model can be obtained fairly easily. It is therefore convenient to use a linear, quadratic or some other manageable type of objective function. It is likely that the preference function[1] of the relevant decision maker is not that simple. It may also involve fixed or semi-fixed targets or it may involve a system of priorities. Therefore, it is convenient to have the objective function of the model approximate that part of the decision maker's preference function which can be approximated by some manageable function and to handle other aspects of the preference function in some other way.

The activities of an activity analysis model

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[1]In this section in order to minimize confusion the term <u>preference function</u> will be reserved for the objective function of the institution's decision maker. The term <u>objective function</u> will refer to the objective function of the <u>model</u> of the institution.

provide the means by which the model approximates the production (and other) functions relevant to the educational institution. When there are only a few different ways of producing a given bundle of outputs and only a few different output combinations that can be produced from a given bundle of inputs, it may be possible to include an activity for each of these different methods of production. If the production function is not only continuous but also has continuous first derivatives and thus more closely resembles those described in many economics textbooks, the activities in the model are chosen to merely approximate the institution's production function.

Besides approximating the production function and other functions related to the institution, the activities included in a model may often be chosen so as to incorporate non-technological restrictions on input combinations, output combinations or input-output combinations.[1] These restrictions may be self-imposed and thus are properly called targets or they may be imposed by decision makers other than the one whose preference function is being

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[1]The sort of restriction most readily handled in this fashion are those which are homogeneous in the inputs and outputs and thus have the form $\sum_i W_i I_i + \sum_j U_j O_j = (, \leqq, \text{or} \geqq)\ 0$, where the $W_i$s and $u_j$ specify the limiting combinations which are permitted.

maximized.

It is convenient to consider the restrictions of activity analysis models as being divided into four groups three of which correspond fairly closely to the three groups of restrictions in problem 4.1.

The first group of constraints consists of those which insure that the output levels are at least as great as their target levels. Three classes of targets, and therefore three sub-groups of constraints, can be distinguished. The first class of targets consists of fixed targets. These targets ordinarily give rise to constraints. A second class consists of those flexible targets which are arguments of the model's objective function. Since these targets ordinarily give rise to constraints of the form

$$A_{fin}^{flex}\, x - y_{fin}^{flex} \geqq 0$$

it may be possible to eliminate them by substituting $A_{fin}^{flex}\, x$ for $y_{fin}^{flex}$ in the objective function. On the other hand it may often be convenient to keep restrictions of this form. A third class consists of those targets (if any) which do not fall into either category. There may be target levels which are arguments of the decision maker's preference function but which are neither fixed nor capable of being handled very well by making them arguments of the model's objective function. This sort of situation could arise if the decision maker specified a list of desired target levels $d_i$ $(i = 1, \ldots n)$ for each of several variables $y_i$ $(i = 1, \ldots n)$ and wanted to maximize K where K is the largest integer such that

$$\sum_{i=1}^{K} (y_i - d_i)^2 = 0.$$

The second group of constraints consists of those which insure that the rate of use of intermediate products does not exceed the rate of production of intermediate products. Since they are homogeneous constraints they can be eliminated but it may be convenient not to eliminate them.[1]

The third group of constraints consists of those which insure that the rate of use of primary factors or resources does not exceed the rate of supply.

A fourth group of constraints has no counterpart in problem 1. These are the constraints, other than resource availability constraints, imposed by persons, groups, or institutions other than the one whose preference function is being maximized. These groups may impose constraints on the amounts of resources which are available but these constraints have already been accounted for in the preceding

[1]The models constructed by Plessner and Fox (33) included virtually no intermediate product constraints.

group of constraints. In addition they may impose restrictions on output levels, on input combinations, on output combinations, or on input-output combinations.

To the extent that these restrictions are restrictions on input ratios, output ratios, or on input-output ratios no additional constraints need be added to the model. Instead the satisfaction of these restrictions may be guaranteed by not including in the model any activity vectors which violate these constraints. Other sorts of restrictions may necessitate the use of additional constraints.

# VI. A MODEL OF A DEPARTMENT

A relatively simple example may illustrate much of what has been discussed up to this point, and may also provide some indication of the usefulness (and perhaps the limitations) of this approach. In order to be somewhat concrete it will be assumed that the educational unit upon which this example is based is an economics department which is one of several departments in one of the colleges of a state supported university. It will be assumed that the department is engaged in research, undergraduate teaching and graduate teaching. In the production of its outputs it uses some resources, such as teaching budget funds, office space, and classroom space, which could readily be used by other departments in the university. It uses some resources, such as the services of its faculty members, the time of its students, and its research funds, which could not as readily be diverted to other departments within the university. It also uses certain inputs, such as the instruction which its students receive from other departments, which are outputs of other departments.

Only a very complicated formulation could hope to include all of the features which might be desired in the model of an individual department. In order to keep the problem manageable several simplifications will be made.

A model of a department could be designed to find the optimum levels of both inputs and outputs. This sort of model would be relevant for some of the decisions which are made about educational institutions. A state legislator, or anyone else who is concerned with the amounts of resources to allocate to an educational institution, would need to take that sort of view of the matter. University officials who argue for larger university budgets must be doing so, at least implicitly, on the basis of a model which allows both outputs and inputs to be variable. For some purposes, however, it may be realistic to assume that some of the input levels can not easily be increased. The departmental model to be presented here will treat some of the available input levels as being fixed and will limit the arguments of the departmental preference function primarily to output levels.

If only one educational product is considered, there are usually at least two sorts of changes that could be made. One way of changing the output is to change the level or rate of output. The other way involves improving or worsening the product. In a very comprehensive model both sorts of changes should be allowed. However, both the structural part of the model and the preference function are more difficult to formulate if both sorts of change are allowed. If some rating system existed which could provide a way of weighting each of the variants of a product, formulation of the preference function would not be too difficult. Unfortunately, many of the rating systems which exist (or might be invented) are at best only ordinal. Cartter (10) has rated the effectiveness of the economics graduate programs offered by several universities. If an economics department could choose among producing (a) two Ph.D's like those granted by University X, (b) one like those granted by University Y, or (c) three like those granted by University Z, it is not obvious that these alternatives should always be ordered (a), (b), (c) even if Cartter's ratings of the respective graduate programs had been 1.50, 2.50 and 0.75.

Production of most educational outputs does not occur instantaneously but requires a certain amount, usually several years, of time. Thus some of the decisions which will affect output levels in a given year must often be made several years in advance. As a result these decisions must often be made on the basis of less than complete information. Thus a completely realistic model of an educational institution should treat both time and uncertainty explicitly. However, the departmental model to be presented will not give them explicit treatment. Instead the model will assume that decisions are made in the face of essentially complete certainty. The model will also be essentially static.

Many educational institutions are engaged in teaching and research during the summer months as well as during the rest of the year. The model to be presented here will not be concerned with the summer months.

The outputs to be considered in the model were mentioned in a general way at the start of this section. A more detailed description of the outputs is presented in Table 52.

There are several groups which might influence the preference function of the sort of department outlined above. These groups could include (a) higher level decision makers in the university or in the state government, (b) persons, firms, or agencies who supply research funds, (c) students, and (d) faculty members. The influence of each of these groups upon a department is not transmitted exclusively through its influence on the departments' preference function. Each of these groups supplies a portion of the inputs used by the department. Presumably the quantity of inputs which each group is willing to supply depends at least in part on its evaluation of the compensation (including non-monetary compensation) which it receives in return. Faculty members may seek employment elsewhere if they become too dissatisfied with the combination of workload, pay, and environment which a particular department offers them. Potential graduate students may tend to seek admission to other universities (or departments) if the graduate program offered by the university in a particular discipline does not seem adequate to them. Suppliers of research funds may reduce the size of the grants made to a department if they expect results which from their point of view are deficient in quantity or quality. The amount of resources which college deans and other higher level educational decision makers supply to a department may depend on the types and amounts of outputs which they expect that the department will produce. Thus each of the groups supplying inputs is effectively imposing one or more constraints on the department.

Since each group's influence may be regarded as being felt via constraints on the department there is no clear-cut justification for allowing any of them to influence the department's preference function. However, for the purposes of this study it will be assumed that the preferences of higher level decision makers in the university and in the state government have the greatest influence on the department's preference function. This assumption is motivated in part by a desire to give more explicit treatment in later chapters to the resource allocation decisions of college deans and other university officials.

It will be assumed that the state's preference function, if it could be known, would tend to give larger weights (or higher priorities) to those programs which serve the state, its people and its industries. One goal which seems to enjoy a rather high priority in most states is the undergraduate education of state residents. Inasmuch as the residence and high school record requirements are often specified in some detail by state statutes or by regulations created by its board of regents, it appears that the intent is (apart from the effect of periodic, and usually minor, tuition and other changes) to largely allow the private demand for education to regulate undergraduate enrollments and thus the rate of Bachelor's degree production. An English economist, Mark Blaug (5), has observed that as the private demand-for-education curve shifts to the right over time, the educational system in the United States tends to shift its supply curve to the right so as to maintain the same "price" and thus keep excess demand for education (at that "price") nearly equal to zero. The "price" variable which Blaug uses is the rate of return on bonds divided by the private rate of return on education.

Other goals also seem to receive some emphasis in most states. A desire to support its economic and social growth is likely to cause the state to give greater weights to those teaching, research, and extension programs which train persons for jobs more or less peculiar to that state, which study the productive, economic, and social processes of the state, or which provide technical assistance and information to the state's production and community groups. Although the motives are less clear, many states also attach positive weights to some outputs which (a) support regional, national, or international goals, (b) benefit other states, other regions, or other nations, or (c) contribute to the maintenance and development of scientific (and other) disciplines.

If the economics department's preference function is consistent with these goals it may involve both fixed and flexible targets. The fixed targets would seem to be most relevant for undergraduate output levels. The levels at which these targets are set depend upon the actions of students. Some other outputs can be subject to flexible target levels. It will be assumed that the preference function is approximately linear in the flexible targets.

The fixed target levels which are to be used are shown in Table 62. The weights associated with the flexible targets are presented in Table 63. The target levels associated with outputs are all non-positive.[1]

Some of the inputs used by the economics department are regarded as being specific to that particular department. It will be supposed that the department has been authorized twenty-five faculty positions, one of which is occupied by the department chairman. He is assumed to devote half of

[1]The choice of non-positive values rather than non-negative values was mostly just a matter of convenience. It is somewhat easier to formulate and check a model if one type of inequality is chosen for all constraints. For maximizing linear programs it has become conventional to use "less than or equal" inequalities.

his effort to administrative functions which affect the department as a whole and which can not readily be allocated to particular outputs or activities. It is assumed that the authorized positions are distributed among three broad areas of interest or specialization within the economics discipline.

The department is assumed to have sources of research funds which support the areas of specialization represented by its faculty.

The department is also assumed able to attract graduate students interested in obtaining Master's or Ph.D. degrees in economics. It is assumed that certain numbers of these students are willing and able, if necessary, to provide their own support. They may have savings which they would be willing to invest in their graduate education. They may already be employed by, or be able to obtain employment with, other academic or service departments of the university. They may have, or be able to obtain, graduate fellowships. They may be employed, or can obtain employment, in the local community. Other students may be attracted by offering them research or teaching assistantships or by offering them employment as instructors.

Table 54, (commodities[1] e46 through e57) describes these inputs and Table 62 indicates the amounts of each of these inputs assumed available to the department.

Some of the inputs used by the department could readily be used by other departments in the university. The amounts of these inputs which are made available to the economics department depend upon the allocation decisions made by the college dean and other university officials in the same way that the amounts of other inputs which are made available depend upon decisions made by students, suppliers of research funds, and others.

The size of the departments' teaching budget is assumed to be decided by the college dean. The budget concept used in this model differs somewhat from the concept ordinarily used by educators. First, truly fixed liabilities are not included in the budget as used in the model. The major liabilities which are considered fixed for the purposes of this model are one-half of the department chairman's salary and the salary for limited-secretarial assistance for the chairman. If the department chairman's

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[1]The term "commodity" may sound a jarring note in this context. It is used here in the comprehensive sense of Koopmans (28, pp. 33-97) who classifies all the products into final commodities, intermediate commodities, and primary commodities.

contribution to the outputs of the department had been considered more explicitly the truly fixed liabilities would be nearly zero.

Secondly, even though the department's faculty is assumed to be fixed in number, the teaching budget need not cover the entire amount of faculty salaries nor any other fixed proportion of faculty salaries. The proportion of faculty salaries included in the teaching budget is determined endogenously. At least two factors which tend to justify, in practice, the "charging" of a certain proportion of each faculty member's salary to a given budget are not relevant here. These factors are uncertainty about the amounts of project research funds that will be forthcoming and the necessity to fulfill contractual obligations made to faculty members. By charging all faculty salaries to budgets whose sizes are known with certainty, it is possible to insure that all faculty salaries will be paid. The first factor, uncertainty, is ignored in this model. The second factor is handled by including other restrictions which insure that faculty members are paid.

Office space and classroom use are included among the inputs allocated to the department by the college dean.

Instruction furnished to economics graduate students by supporting departments and teaching resources furnished to the economics department by supporting departments are also regarded in this model as being inputs allocated to the department by the college dean.

Table 54 (commodities e38 through e45) describes the inputs allocated to the department by the college dean. Table 62 indicates the amounts of each of these inputs which the college dean is assumed to have allocated to the department.

Table 53 describes the intermediate products considered by the model.

It has been assumed that the numbers of faculty members of various specializations or types which the department is authorized to employ is fixed. It seems safe to presume that the supply function which the department faces for faculty members of the ith type includes as arguments not only salary levels (plus fringe benefits) but also factors such as the amounts of teaching and research required, and the office space per faculty member. It is also likely to include as arguments several variables over which the department chairman has no control.

Let the ith supply function be written

$q_i = f_i(z_i)$ , where $z_i$ is the vector of arguments

which the department chairman can control. Let $\bar{q}_i$ be the number authorized of ith type faculty and define the set

$$Q_i = \{ q_i | \; q_i \geqq \bar{q}_i \}.$$

The department chairman is thus limited to $z_i$ vectors which belong to the set

$$z_i = f^{-1}(Q_i) = \left\{ z_i \mid f_i(z_i) \geqq \bar{q}_i \right\}.$$

For the purposes of the present study $z_i$ will be assumed to have seven components; (1) salary (and benefits) paid from the teaching budget, (2) salary (and benefits) paid from a research budget, (3) proportion of faculty member's time allocated to research, (4) amount of undergraduate teaching per year per faculty member, (5) amount of graduate teaching per year per faculty member, (6) amount of office space per faculty member, (7) amount of time spent on academic or professional committees and other "public service" functions per year per faculty member. It will be assumed that the terms on which research funds have been obtained require that the proportion of a faculty members time allocated to research supported by a particular research budget must be at least as great as the proportion of his salary which is paid out of that budget. It will also be assumed that the amount of "public service" time required per faculty member is proportional to the amount of time spent in teaching functions per faculty member (one-eighth of such time was the figure chosen.) Other restrictions may be imposed by the college dean or other decision makers. Define the set $Q'_i$ as the set of vectors which both belong to $Q_i$ and satisfy the restrictions imposed by other decision makers.

If $Q_i'$ is convex, elements $z_i^j$ of $Q_i'$ can be used to construct activity vectors for the model. Several activity vectors which for the purposes of this model are assumed feasible are presented in Table 61. For the particular activities which have been chosen the total salary demanded by faculty members is independent of the amounts of various services rendered. For these activities the total workload per faculty members "adds up" to the equivalent of a workload of 12 "contact" hours per week. Activities satisfying these two requirements were chosen only to make the model easier to understand.

It is assumed that any faculty member is capable of teaching economic principles and that any faculty member is also as willing to supply one unit of teaching time for economic principles teaching as to supply one unit for undergraduate teaching in the faculty member's specialty. These assumptions are incorporated into the model by the first three "transfer" activities included in Table 61. These assumptions could have been incorporated by means of activities having the same form as the other faculty allocation activities, but to do this would have required the use of six activities rather than three.

The fourth "transfer" activity in Table 61 allows those "specialty one" faculty members who enjoy project research support to participate in non-project research. As in the case of the other "transfer" activites its use was motivated by the fact that its use permitted using fewer activities.

It will be assumed that as many secretarial and other clerical personnel as are desired can be employed at some "going wage" rate. The specific assumptions about the wage rate, office space requirements, and so forth for secretarial and clerical personnel are incorporated in the activities shown in Table 60.

For the purposes of the present study it is assumed that additional (beyond those ready, willing, and able to obtain their own support) graduate students can be obtained by appointing them as one-half time research assistants, teaching assistants, or as instructors. It is assumed that as many students as are desired can be obtained at the "going rate" for the various types of appointments. The activities which incorporate these assumptions are presented in Table 59. The first activity (e43) is designed to allow the use, if necessary, of instructors as teaching assistants. Eight of the other activities (e44, e45, e46, e47, e48, e51, e53, and e56) are "pure" appointments in the sense that they represent appointments as research assistants, teaching assistants or instructors. The other six are weighted averages of these eight except that the number of years required to obtain a degree (the inverse of the absolute value of the elements in rows e51 and e52) are less than the same weighted averages of the numbers of years required for "pure" appointments. This is to recognize the fact that a student who teaches during the time that he studies for his Ph.D. is likely to require longer to complete the degree than if he were on a research assistantship for the last year or so of his program. It was also assumed that a student who is on a research assistantship during his entire Ph.D. program is likely to be able to finish sooner than if he had taught during some portion of his program.

The next activities to be considered are teaching activities. The most important coefficients in each of these activity vectors are those which specify the marginal class sizes. Several factors may affect class sizes. The first factor affecting class sizes is the unit of measurement of faculty teaching inputs. Since this unit has been defined as the amount of faculty teaching inputs needed to teach one section, the class sizes used help to complete the unit of measure adopted for teaching inputs. Snodgrass (36, p. 322) has estimated that one section of a certain 3 credit hour, semester length course would, if adequately taught, require 230 hours of faculty time plus 7 additional hours for each student enrolled in the course. Since the faculty inputs required per section of that course and, presumably, any course depend upon the class size, specifying the class sizes to be permitted for planning purposes helps to establish the size of a unit of teaching inputs.

A second factor affecting permissible class sizes is the effect of class size on the quality of instruction received by the students enrolled in the courses. The class sizes which will allow adequate instruction are influenced by many factors including course content and the nature of available classrooms.

Scheduling and coordination considerations also may affect class sizes. On the one hand, as enrollment increases it may be necessary to increase the number of sections of a course more rapidly than would otherwise be required in order to insure that the course is offered at enough different times during the year so that all students required to take the course can fit it into their class schedules. On the other hand, even if enrollment is low some courses may need to be taught at least annually or biennially in order to maintain an undergraduate curriculum or a graduate degree program. In such a case average class sizes may be rather low and the number of sections offered per year need not increase as fast in response to increased enrollment as would otherwise be required.

Two activities are included in the model for each of the (three) undergraduate course areas, other than economic principles. One of each of these pairs of activities permits all of the teaching to be done by faculty members. The other member of each pair of activities allows a portion of the teaching to be done by graduate instructors under the supervision of faculty members teaching in the same area.

Faculty administrative inputs necessary to supervise the teaching program are not treated separately. Part of these are furnished by the department chairman and thus under the approach adopted here are not accounted for at all. Those which are contributed by other faculty members are taken into account by adding 0.1 units to the faculty teaching inputs required per section. Thus for each of these activities the faculty input coefficient plus the graduate instructor input coefficient equals 1.1 rather than 1.0.

One activity was included in the model for each of the three graduate course areas. It was assumed that administrative inputs per graduate section must be 0.05 units. Thus the faculty inputs required per section equal 1.05 units of teaching inputs instead of the one unit that might be expected.

For economics principles teaching it was assumed that scheduling and other coordination difficulties preclude average class sizes greater than one hundred, but that otherwise large lecture sections of up to 220 could be permitted. Thus if the $\alpha_i$'s are the class sizes associated with six ways of teaching economics principles and E is the enrollment in principles, two of the constraints involving the teaching activities for economics principles are given by

$$\sum_{i=1}^{6} \alpha_i x_i \geqq E$$

$$\sum_{i=1}^{6} x_i \geqq \frac{E}{100}$$

$$X_i \geqq 0, \ i = 1, 2 \ldots 6$$

Six activities were originally considered. Three of these involved class sizes of 220. The six activities actually included in the model were based on the six efficient basic solutions to the small linear program defined above. For the original activities faculty administrative requirements were set at 0.1 unit for sections smaller than 60 and at 0.25 unit for larger sections. The larger sections would involve the use of the services of teaching assistants.

Activities were included in the model for thesis supervision and the advising of students. Although these activities have the same form as other teaching activities the "class size" coefficients have somewhat different meanings. In the case of advising activities the "class size" coefficient represents the number of advisees which a faculty member could advise if his classroom teaching assignments were reduced by one section per quarter. For thesis supervision the "class size" coefficient is one third times the number of hours of thesis credit which could be supervised per unit of teaching inputs. The input coefficients for the thesis supervision activities indicate the relative contributions by each of the several groups of faculty members to thesis supervision (and to other functions performed by students' supervisary committees).

Tables 57 and 58 present the teaching activities included in the model. Table 57 presents the undergraduate teaching and advising activities. Table 58 presents the graduate teaching, advising, and thesis supervision activities.

Degree producing activities will be considered next. Although an undergraduate activity has been included in this group of activities, it is not, strictly speaking, a degree activity since it is concerned only with the instruction and advising which an economics major receives from the economics department.

Since most economics majors (both graduate and undergraduate) are not faced with the task of completing a rigid list of courses it might seem that several activities should be included for each type of degree. However, much of this freedom resides with the student and not with the department. Furthermore, that which resides with the department seems more likely to be used for the student's benefit rather than to reduce the cost of a degree. Thus, at least for this model, determining the average curriculum for a particular degree is not an optimization problem but a prediction problem.

It is assumed that the required numbers of credit hours are 30, 45, and 63 for an undergraduate major, a Master's degree, and a Ph.D. degree (for a person who already has a Master's degree), respectively. For the model these requirements have been set at 36, 51, and 81, respectively, to reflect the fact that students often take more than the minimum numbers of courses required and to reflect the fact that attrition rates are not zero. The instructional input requirements for the activities are specified in terms of numbers of (3 credit hour) courses. Table 56 presents the degree activities.

The only activities remaining to be discussed are research activities. The research output per faculty man year undoubtedly depends upon the amounts of other research inputs which are combined with the faculty. Several activities are included for each area of research in order to allow differing amounts of other inputs to be combined with each faculty research year. Table 55 presents these activities.

The solution vector obtained by maximizing the objective function is presented in Table 64. While the solution is not unique, the final output levels are not very sensitive to minor changes in the objective function or to minor changes in the b vector.

The dual solution vector is presented in Table 65. The solution to the dual is quite sensitive to minor changes in the b vector.

The sensitivity of the dual solution is, of course, due to the fact that the model does not allow much substitution among certain of the inputs. In many cases it is realistic to believe that only limited substitution is possible, but it may be that the model presented in this chapter understates the substitution that could be permitted.

The model allows no substitution on the "supply" side (i.e. the b vector is fixed). Thus it is possible that the model also understates the substitution that could occur on the supply side. In some cases suppliers of resources may react to the "shadow prices" of these resources by supplying more of resources for which the shadow prices seem to them to be "too high" and less of resources for which the shadow prices seem to be "too low". This sort of reaction cannot be expected, however, unless the department's objective function is consistent with that of the particular resource supplies being considered.

For the example considered in this chapter the department's objective function was assumed to be consistent with that of the college dean. Therefore, it may be that the college dean would (or at least could) use the "shadow prices" on the resources which he supplies to guide his allocation of these resources. This possibility along with other aspects of the decision making process relevant to a college will be discussed in the next chapter.

# VII. A MODEL OF A SMALL COLLEGE

Most of the productive processes relevant to a college model are supervised by the various departments in the college. One approach to the construction of a model of a college is thus to first construct models of its individual departments. The economics department of a university is likely to be found in a college of the Arts and Sciences type. Such a college may include twenty or more departments. To attempt to consider each of these departments, even if in no greater detail than the economics department was considered, would result in a very large college model. If such a model were designed to aid in the management of an actual college and if it could be based on a careful investigation of each of these departments, construction of such a model might be justified. However, for the purposes of this study, a smaller model can both serve to illustrate some of the features which a larger model might have and to provide some understanding of the source of some of the constraints found in the departmental model presented earlier. The college model to be presented here will be based on a hypothetical college which is presumed to include only an economics department and the departments (A and B) which furnish relatively large amount of (graduate level) service teaching to economics department graduate students.

The relationships assumed to exist between the activity levels and net input (of commodities) levels can, for the ith department, be expressed as $A_i X_i = y_i$. The coefficients of the matrix $A_a$ (department A) and $A_b$ (department b) can be found in Appendices A and B, respectively.

Tables 3, 4, and 5 list and describe those commodities which are considered to be final products, intermediate products, and primary products, respectively from the point of view of department A. Tables 16, 17, and 18 list and describe those commodities which are considered final products, intermediate products, and primary products, respectively, from the point of view of department B.

Tables 6 through 11 present the activities included in the model of department A. Tables 19 through 24 present the activities included in the model of department B.

Not all of the outputs of a college need be regarded as outputs of one of its departments. One such output which will be considered is the Bachelor's degree. It may be inappropriate to consider Bachelor's degrees as outputs of the individual departments since only a fraction of the training which a student must receive in order to be awarded that degree occurs within his major department. It may be relevant, however, to consider Bachelor's degrees as outputs of a college since (at least for a college of the Arts and Sciences type) most of the training required occurs within the college.

It will be assumed that the major variables (such as entrance requirements and tuition rates) which influence the numbers of persons seeking B.S. degrees are set by higher level decision makers and that the college will educate any undergraduate student who chooses to enroll and who maintains a satisfactory grade average. It will also be assumed that the college does not influence the distribution of undergraduate students among the various majors offered by the college. This is equivalent to adopting fixed target levels for B.S. degree outputs. The model could include an activity for each different major offered by the college. The activity level vector Z associated with these activities would be required to satisfy the restriction $DZ = d$ where d is a vector composed of the rates at which B.S. degrees are to be awarded in each of the various majors and $D^{-1}$ is a diagonal matrix composed of the levels at which each of the B.S. degree producing activities must be operated in order to produce one B.S. degree of each type per year. For the purposes of the model to be presented here the activity level vector and the corresponding activity vectors will be collapsed into a scalar, $Z_T$, and a single activity vector respectively by letting $1'Z = X_{c48}$ and $1'd = b_1$.

This effectively means that the product "Bachelor's degrees" produced by the college now consists of $\pi_i$ percent of ith major Bachelors degrees. The coefficients for the

Bachelor's degree producing activity are shown in Table 29, Appendix C.

Although the actual method of handling this matter would depend upon the organization of the university and upon the physical arrangement of the campus buildings, it will be assumed that the college dean is assigned a block of small classrooms for use by the teaching activities of his college. Even though the college dean has control of a classroom for 168 hours a week, it is clear that each classroom can not yield 168 classroom units per quarter[1] (i.e. one classroom is not sufficient to allow the conducting of 56 sections per quarter of courses meeting 3 hours each per week). A more feasible rate of classroom use would seem to be about 30 hours per week, but this certainly cannot be taken as an upper bound.

For the purposes of this model it will be assumed that there is some slight cost associated with increasing the rate of classroom use per classroom. It will be assumed that for rates of use up to 25 hours per week (75 units per year) these costs are already accounted for in the teaching activities of the college. Beyond this rate of use it will be assumed that coordination problems are encountered. That

[1]Classroom units defined in Tables 5, 18, or 54.

is, classes may need on the average to be scheduled farther away from the offices of the department supplying the instructor, classes may need to be scheduled at other than prime teaching times, or classes may have (on the average) to be conducted in classrooms less suited to the method of presentation most useful for the subject matter. This will be assumed to increase slightly the average salary demanded by staff members. It is reasonable to believe janitorial and maintenance costs per classroom may increase slightly. Finally the cost of achieving a feasible allocation of classroom use among the teaching activities of the college can be expected to increase slightly.

This situation will be incorporated in the model by including a classroom allocation "department" in the model. This department will be assumed to have the function not only of allocating periods of classroom use but also of compensating other elements of the university for increased costs incurred as a result of choosing use rates in excess of 25 hours per week.

The activities of the classroom allocation department are shown in Table 49.

There are several ways of arranging the elements of the model. One way would be to group the constraints into three groups corresponding to those commodities which are regarded as final products, intermediate products, and primary products from the point of view of the college. Although this method of presenting the model will not be especially useful for later sections, this will be the first way of presenting the model.

With the exception of the elements of the "c" and "b" vectors, all of the coefficients of the model have already been presented. They will not be presented again. Instead cross references will be provided between the commodity and activity numbers used in the college model and the commodity and activity numbers used in the departmental models. These cross references are to be found in Tables 30 through 32 and Table 33, respectively.

The restrictions faced by the college are shown in Table 34. No restriction is placed upon the amounts of instruction which the college can obtain from departments in other colleges. Instead an opportunity cost of 0.12 per instructional unit is assigned. It is assumed that the impact of this instructional load is fairly widely distributed among the departments in the rest of the university. (If this is the case a constant opportunity cost may not be too bad an approximation. If the impact were not widely distributed, as would be the case if one department supplied all of this instruction the opportunity cost per unit would probably depend upon the number of units of instruction supplied.) Teaching resources supplied by other colleges are treated in the same manner. An opportunity cost of 1.5 per unit of teaching is assumed.

Table 35 presents the objective function weights associated with flexible targets. Although it is possible that these weights could have been derived from the college dean's own preference function, it will be assumed that they are based on guidelines or weights supplied by the university president. The university president may have specified the weights to be associated with each individual output. On the other hand, he may have established weights on contributions made to the states' economy, to ameliorating the social problems of the state, to national scientific disciplines, and to other objectives, and asked the college dean to use these weights as a basis for computing the weights for each sort of output produced by the college (with the exception of those outputs whose levels have been made fixed targets).

To facilitate discussion in the following sections, the model just described will be called version one and will be represented by 7.1.

$$(7.1)\quad \max C_c' x_c$$

$$x_c \in X_c$$

$$X_c = \left\{ x_c \mid A_c x_c = y_c \leqq b_c,\ x_c \geqq 0 \right\}^{1}$$

[1]Not all of the restrictions were presented in the model as less than or equal restrictions. It does no harm to discuss the model as though it had the form of 7.1 since if it were really necessary to do so the model could be put in the form of 7.1.

For most of the discussion which follows it will be convenient to have a version of the model which has no non-departmental activities. In the discussion presented above a separate "department" was "created" to manage the allocation of classrooms. The only other activity in the model which is not an activity of an academic department is the activity associated with Bachelors' degree production (column 48 of $A_c$). The main reason for including it in version one was to help clarify the effect of the assumptions which were made about the determinants of the levels of undergraduate outputs. The assumptions which were made allow this activity to be easily eliminated from the model.

Version one can be rewritten as

$$\max \left\{(C_c^-)' \ (x_c^-) + (0) \ x_{c48}\right\}$$

subject to

$$A_c^- \ x_c^- \leq b_c - A_{c48} x_{c48}$$

where $A_{c48}$ is the 48th column of $A_c$,

$A_c^-$ is $A_c$ after removing the 48th column,

$C_c^-$ is $C_c$ after removing the 48th element of $C_c$,

$x_{c48}$ is the 48th element of $x_c$,

and $x_c^-$ is $x_c$ after removal of $x_{c48}$.

Since it is required that $x_{c48} = 250$, version one can be rewritten as

$$\max c_c^- x_c^-$$

$$x_c^- \in X_c^-$$

$$X_c^- = \left\{ x_c^- \mid A_c^- x_c^- \leqq b_c^- ,\; x_c^- \geqq 0 \right\}$$

where $b_c^- = b_c - (250)A_{c48}$.

Version two could then be obtained by rearranging the elements of $A_c^-$ so that the matrix A which results has the form

$$A = \begin{bmatrix} \bar{A}_a & \bar{A}_b & \bar{A}_d & \bar{A}_e \\ \underline{A}_a & & O & \\ & \underline{A}_b & & \\ & O & \underline{A}_d & \\ & & & \underline{A}_e \end{bmatrix}$$

When put in this form the first constraints in the model are those which affect more than one department.[1] The next group of constraints includes those affecting only department A, the third group of constraints includes those affecting only department B, and so forth. The first group of activities

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[1]Although this will be spelled out in more detail later it may be worthwhile to mention these common constraints here. The common constraints relate to teaching budget funds, space resources (office space and classroom use) and to those commodities (teaching services and instruction) which are produced by one department partly for use by one or more of the other departments in the college.

(columns) includes the activities of department A, the second group includes the activities of department B, and so on. The vectors $c_{\bar{c}}$, $x_{\bar{c}}$ and $b_{\bar{c}}$ are subjected to the same sort of rearrangement. The resulting vectors have the forms

$$c = \begin{bmatrix} c_a \\ c_b \\ c_d \\ c_e \end{bmatrix} \qquad x = \begin{bmatrix} x_a \\ x_b \\ x_d \\ x_e \end{bmatrix}, \quad \text{and } b = \begin{bmatrix} \bar{b} \\ \underline{b}_a \\ \underline{b}_b \\ \underline{b}_d \\ \underline{b}_e \end{bmatrix} .$$

The easiest way to obtain version two of the model is to start directly from the departmental models. Tables 36 and 37 provide cross references between the commodity and activity numbers, used in version two of the college model and the commodity and activity numbers used in the models of the individual department. Table 38 presents the b vector relevant to version two. Table 39 presents the objective function weights for version two.

Version two will be represented by 7.2.

$$(7.2) \quad \max_{x \in X} c'x$$

$$X = \{x \mid Ax = y \leqq b \, , \; x \geqq 0\}$$

This section will examine the feasibility of alternative decision making arrangements. It will deal with one arrangement under which the college dean selects the level at which each activity is to be operated, and with other arrangements under which the college dean influences the activity levels less directly. The first arrangement will be called complete centralization,[1] and the other arrangements will be called partial decentralization.

Not too much can be said about which decision making arrangement is best since many of the factors which are relevant to that sort of determination will not be treated very carefully.[2] It will be assumed however, that the ith department has more complete information about the elements of $\overline{A}_i$, $\underline{A}_i$, and $\underline{b}_i$. It is clear that this should definitely be the case for $\overline{A}_i$, and $\underline{A}_i$ since the coefficients in these matrices are related to processes supervised by the department. It is equally clear that this should be so for those elements of $\underline{b}_i$ which are associated with input restrictions faced only by department i. It is not as clear that this should be so for those elements of $\underline{b}_i$ which are associated with output restrictions. In the model presented above these elements (i.e. those elements of $\underline{b}_i$ associated with output restrictions) depend upon undergraduate enrollments

[1]The first arrangement represents complete centralization only with respect to those activity level decisions explicitly included in the model.

[2]Some of these factors are discussed in (30).

within the college and university and upon graduate enrollment in the rest of the university.

The information about enrollments is likely to be known by the college dean, but it is also the sort of information that is widely disseminated. The department chairman could probably estimate the output levels required by these enrollments at least as easily and accurately as the college dean could. It was already assumed above that the college dean formulates the weights for the college objective function.[1]

A decision making scheme will be considered effective if, under that scheme, the college dean can be assured that activity level decisions will be reached which will ensure (a) satisfaction of all commodity constraints (both those impared by suppliers of resources and those which result from the establishment of fixed targets) and (b) (subject to the commodity constraints) maximization of the college's objective function. For each scheme, the college dean will be assumed to be using two sets of instruments. It will be assumed that he uses one set of instruments to guide the operation of his college, and that he uses the other set in the planning phase in order to determine the optimum values for the first set of instruments. Thus the second set of

[1]This assumption could easily be modified. This will be done in Chapter 8.

instruments serves to direct an information gathering process. These two sets of instruments may be identical, but they need not be. Since educational institutions are planned institutions in much the same sense that socialist and certain other types of economics are planned, it may make sense to use one set of instruments to form the plan and another to guide its fulfillment.

Since two sets of instruments are involved, it is convenient to divide the discussion into two parts. In the first part, different sets of operational control instruments will be considered effective if there exist values for that set of instruments which will insure that the constraints in the college model are satisfied and that, subject to those constraints, the college objective function will be maximized. In the second part of the discussion, alternative sets of information gathering instruments will be examined. These sets of instruments will be considered effective for a given set of operational control instruments if they can be used in such a way as to insure that optimum values can be obtained for that set of operational control instruments.

A decision making scheme will thus be considered effective if its set of operational control instruments is effective and its set of information gathering instruments is effective for the operational control instruments chosen.

Under complete centralization the college dean would be expected to use the elements of the vector $x_c$ (or in the case of version two, the vector x) as his set of operational control instruments. This set of instruments is, by definition, an effective set of operational control instruments if an effective set exists.[1]

Under complete decentralization it might be expected that the college dean's set of operational control instruments would include only prices. That is, he would not set any activity levels nor would he establish any departmental input or output quotas. The completely decentralized case will not be considered here; the discussion will be confined to schemes involving only partial decentralization.

One set of operational control instruments to be considered will include one instrument for each activity in version two of the model and one instrument for each constraint in version two which affects more than one department. All of the instruments would be prices.

If this set of instruments were adopted, the college dean would select values for each of the elements in the vectors

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[1]It will be assumed throughout the rest of this chapter that both versions of the model are solvable. That is, it will be assumed that there exists at least one feasible vector $x_c^*$ such that

$$c_c' x_c^* = \max_{x_c \in X_c} c_c' x_c$$

$$\rho = \begin{bmatrix} \rho_1 \\ \cdot \\ \cdot \\ \cdot \\ \rho_n \end{bmatrix}$$ [1] and $\pi$, and then ask

the ith department chairman to select activity levels which would solve 7.3.

$$(7.3) \quad \max_{x_i \in X_i} \left\{ \rho_i' \, x_i - \pi' \bar{y}_i \right\}$$

$$X_i = \left\{ x_i \mid \bar{A}_i \, x_i = \bar{y}_i, \; \underline{A}_i \, x_i \leqq b_i, \; x_i \geqq 0 \right\}$$

The elements of $\rho_i$ are the weights or prices which the ith department chairman would be asked to associate with the various activities of his department; $\pi$ is a vector of weights to be associated with the elements of the vector $\bar{y}_i$, and $\bar{y}_i$ is a vector of the amounts "used" by the ith department of those commodities which also affect at least one other department.

This set of instruments can be considered effective only if there exist vectors $\tilde{\rho}$ and $\tilde{\pi}$ such that any solution to 7.4 must also solve 7.2.

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[1]Throughout the remainder of this chapter, it will be convenient to use the more conventional numerical subscripts rather than alphabetical subscripts.

$$(7.4)\quad \max_{x\in \underline{X}} (\tilde{\rho} - \bar{A}'\tilde{\pi})' x$$

$$\underline{X} = \{x \mid \underline{A}x \leq \underline{b},\ x \leq 0\}$$

where

$$A = \begin{bmatrix} \underline{A}_1 & & & 0 \\ & \underline{A}_2 & & \\ & & \ddots & \\ 0 & & & \underline{A}_n \end{bmatrix} \text{ and } \underline{b} = \begin{bmatrix} \underline{b}_1 \\ \vdots \\ \underline{b}_n \end{bmatrix}.$$

Although in all cases there will exist sets of values for the elements of $\rho$ and $\pi$ which are at least as good, the most obvious values for $\tilde{\rho}$ and $\tilde{\pi}$ are obtained by setting $\tilde{\rho} = c$ and $\tilde{\pi} = \bar{v}^*$.[1] It is clear that if these values are chosen, then any solution to 7.2 is also a solution to 7.4, but it is also clear that some of the solutions to 7.4 may not be solutions to 7.2. This is due to the fact that some of the solutions to 7.4 may not be feasible solutions to 7.2.

There may be particular problems (i.e. particular values for the elements of A, b, c,) for which there would exist values $\tilde{\rho}$ and $\tilde{\pi}$ such that every solution to 7.4 is also a solution to 7.2 but there are also problems for which this is not so.

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[1] $\bar{V}^*$ is that part of the dual solution vector $\bar{V}^* = \begin{bmatrix} \bar{V}^* \\ \underline{V}^* \end{bmatrix}$ which correspond to those constraints affecting more than one department.

Consider a case in which there are two departments, one of which produces an output, say instruction of some sort, which is used by the other department. Suppose further that the resources used to produce this output could also be used in the production of other outputs. As a numerical example consider the problem in which:

$$x_1 = x_1 \ , \ x_2 = \begin{bmatrix} x_{21} \\ x_{21} \end{bmatrix} \ , \ c_1 = 3 \ , \ c_2 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$\overline{A}_1 = 2 \ , \ \underline{A}_1 = 1 \ , \ \overline{A}_2 = [0 \ \ -1], \ \underline{A}_2 = [1 \ 1]$$

$$\overline{b} = 0 \ , \ \underline{b}_1 = 1 \ , \ \underline{b}_2 = 3$$

7.2 would then become:

$$\max 3x_1 + x_{21}$$

subject to

$$\begin{aligned} 2x_1 - x_{22} &\leqq 0 \\ x_1 \qquad &\leqq 1 \\ x_{21} + x_{22} &\leqq 3 \\ x_1, x_{21}, x_{22} &\geqq 0 \end{aligned}$$

7.4 becomes:

$$\max \left\{ (\rho_1 - 2\pi)x_1 + \rho_{21}x_{21} + (\rho_{22} + \pi)x_{22} \right\}$$

subject to

$$\begin{aligned} x_1 &\leqq 1 \\ x_{21} + x_{22} &\leqq 3 \\ x_1, x_{21}, x_{22} &\geqq 0 \end{aligned}$$

The primal and dual solutions to 7.2 are

primal: $x_1 = 1$, $x_{21} = 1$, $x_{22} = 2$

dual: $\overline{v} = 1$, $\underline{v}_1 = 1$, $\underline{v}_2 = 1$

The "best" values for $\rho$ and $\pi$ must satisfy

$$(\rho_1 - 2\pi) > 0$$
$$\rho_{21} = (\rho_{22} + \pi) > 0$$
$$\pi > 0$$

The solutions to 7.4 are given by

$$x_1 = 1$$
$$x_{21} = 3\lambda$$
$$x_{22} = 3(1 - \lambda)$$
$$0 \leqq \lambda \leqq 1$$

Only one of these solutions (the one with $\lambda = 1/3$) also solves 7.2.

An article by Kornai and Lipták (29) suggest an alternative set of operational control instruments. Their results were designed for application to a socialist economy but they are equally applicable here. Their results suggest the use of a set of instruments which includes one price for each activity in the model, one quota for each constraint which affects no more than one department, and four quotas for each constraint which affects more than one department.

If this set of instruments were chosen the college dean would select values for the elements of the vectors

$$\rho = \begin{bmatrix} \rho_1 \\ \rho_2 \\ \vdots \\ \rho_n \end{bmatrix}, \qquad \text{and} \qquad u = \begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_m \end{bmatrix}$$

He would then ask the ith department chairman to select activity levels which would solve 7.5.

$$(7.5)\quad \max_{x_i \in X_i(u_i)} \rho_i' x_i$$

$$X_i(u_i) = \left\{ x_i \;\middle|\; \begin{bmatrix} \overline{A}_i \\ \underline{A}_i \end{bmatrix} x_i \leqq \begin{bmatrix} u_i \\ \underline{b}_i \end{bmatrix}, \; x_i \geqq 0 \right\}.$$

The dean should, if he follows Kornai and Lipták's procedure, set $\rho = c$. Then 7.5 would become

$$(7.6)\quad \max_{x_i \in X_i(u_i)} c_i' x_i$$

The set of control instruments can be considered effective only if there exists at least one vector $u^*$ such that if the vectors $\hat{x}_i(u_i^*)$ solve problems 7.6 then the vector $\hat{x}(u^*) = \begin{bmatrix} \hat{x}_1(u_1^*) \\ \vdots \\ \hat{x}_n(u_n^*) \end{bmatrix}$ will solve 7.2. The existence of such a vector can be shown by construction.[1]

Let $x^* = \begin{bmatrix} x_1^* \\ \vdots \\ x_n^* \end{bmatrix}$ solve 7.2. Let $u_i^* = \overline{A}_i x_i^*$ for $i = 1, 2 \ldots n$. Suppose when $u_i$ is set equal to $u_i^*$ that $\hat{x}_i$ solves the ith version of 7.6. Thus $c_i'\hat{x}_i = \max_{x_i \in X_i(u_i^*)} c_i' x_i \geqq c_i' x_i^*$ since $x_i^*$ necessarily belongs to $X_i(u_i^*)$. Let $\hat{x} = \begin{bmatrix} \hat{x}_1 \\ \vdots \\ \hat{x}_n \end{bmatrix}$.

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[1]For a more comprehensive treatment see (29).

Then $c'\hat{x} = \sum_{i=1}^{n} c_i'\hat{x}_i \geq c_i'x_i^* = c'x^*$. $\hat{x}$ necessarily belongs to X and thus by assumption[1] $c'\hat{x} \leq c'x^*$. Therefore, $\hat{x}$ solves 7.2.

Two sets of operational control instruments have been found effective. Presumably these two sets of instruments could be "mixed" in ways which would yield other effective sets of instruments, but this will not be attempted here.

This section will discuss alternative ways of discovering the best values for the operational control instruments considered above.

One way of obtaining optimum values for these instruments would be for the college dean to solve directly the appropriate version of the college model. That is, he would obtain all of the coefficients in version one or version two, and solve the model as an ordinary linear program. If he chooses the first set of operational control instruments the solution $x_c^*$ or $x^*$ would give the appropriate values for his operational control instruments. If he chooses the second set of operational control instruments he should solve the dual of version two. He can then set $\tilde{\Pi} = \bar{v}^*$ and $\tilde{\rho} = c$ $\left(v^* = \begin{bmatrix} \bar{v}^* \\ \underline{v}^* \end{bmatrix}\right.$ solves the dual of (7.2).)

If he chooses the third set of operational control instruments, he should solve 7.2. The solution vector

[1]The assumption was that $x^*$ solves 7.2 and therefore $c'x^* \geq c'\tilde{x}$ for all $\tilde{x}$ belonging to X.

$x^* = \begin{bmatrix} x_1^* \\ \vdots \\ x_n^* \end{bmatrix}$ can then be used to obtain $u^* = \begin{bmatrix} u_1^* \\ \vdots \\ u_n^* \end{bmatrix}$ by letting $u_i^* = \bar{A}_i x_i^*$.

Since this way of obtaining optimum values for the control instruments involves collecting whatever information may be needed prior to attempting to solve the problem (7.1 or 7.2), it is not immediately obvious what, if any, instruments would be used to help collect the information.

It was assumed earlier that the department chairmen's information about the elements of the matrices $\underline{A}_i$, $\bar{A}_i$ and vectors $\underline{b}_i$ was more complete than that of the college dean. Presumably then, the college dean must obtain whatever information about A and $\underline{b}$ that he uses from the department chairmen. If their (and his) time has alternative uses, he may not want them to take the time needed to tell him (nor will he want to take the time needed to listen to or read their reports and to sort the relevant material from the irrelevant) "everything that they know about their departments". Instead he might ask them to reveal, initially, their $\underline{b}_i$ vectors (the quantitative restrictions found only by the individual departments) and enough columns of A to allow him to obtain a feasible solution (to 7.1 or 7.2).[1]

He could use this information to compute a tentative

[1]The requirement that the department chairmen initially guarantee the dean a feasible solution could be dispensed with by using an appropriate "first stage" procedure.

solution. The elements of the corresponding dual solution vector could be used to guide the search for information which could improve the solution. The process of discovering optimum values for the college dean's control vector would thus become an iterative one.

During any (the Nth say) phase of this process the college dean would use those columns of A which he knows to solve the dual of 7.7.

$$(7.7) \quad \max_{x\langle N\rangle \in X\langle N\rangle} c\langle N\rangle' \, x\langle N\rangle$$

$$X\langle N\rangle = \{x\langle N\rangle \mid A\langle N\rangle x\langle N\rangle \leqq b, \; x\langle N\rangle \geqq 0\}.$$

The $\langle N\rangle$ following A, (and c' and x) merely indicates that portion (i.e. those columns) of A ( and the corresponding elements of c' and x) which are known at the beginning of the Nth phase.

During the second part of the Nth phase the college dean would inform the department chairmen of the elements of c and $v^*\langle N\rangle$ (the solution of the dual of the Nth phase version of 7.7) and ask them to report the technical coefficients (the elements in the column of the A matrix) corresponding to any activity such that $c_j - A_j' \, v^*\langle N\rangle$ is greater than zero.[1,2]

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[1] $c_j$, and $A_j$ refer in this context to individual activities and not to groups of activities.

[2] It is evident the dean would have to emphasize that the activity must (in the case of those activities for which $c_j \neq 0$) produce or use one unit of some commodity. The unit would also need to be defined.

If any columns are reported they are added to $A\langle N\rangle$ to form $A\langle N + 1\rangle$ for the next phase. If none are found the process would be terminated. The solution to the final version of 7.7 could be used to obtain an optimum set of values for the first or third set of operational control instruments. The solution to the dual of the final version of 7.7 could be used to obtain appropriate values for the second set of operational control instruments described.

If the above process were adopted it is apparent that commodity prices are the instruments which are used to guide the information gathering process.

If on the other hand the college dean knew at the beginning of each phase only some rows of the matrix (A b), the solution vector $x^*\langle N\rangle$ would guide the information gathering process. The college dean would ask the department chairmen to report rows of (A b) which correspond to constraints violated by the Nth phase solution $x^*\langle N\rangle$ .[1]

A mixed process could also be used. If the mixed process were adopted, at the beginning of the Nth phase, some rows of (A b) and some columns of A would not be known to the college dean. He would obtain a primal and dual solution based on the information available to him. The department

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[1]If none were found an optimum solution can be computed from the Nth phase version of the problem (as known to the college dean). If some are found they could be used to generate the N + 1th phase version of the problem.

chairmen would then be asked to report any "profitable" activities (columns of A) that they know of and any restrictions (rows of (A b)) which the Nth phase solution violates.[1]

If both the number of rows and the number of columns needed to describe the college are finite (i.e. if the dimensions of A are finite) any of these procedures would terminate after a finite number of phases. The terminal (primal and dual) solutions obtained for any one of them could (barring human error) be used to construct the solution to the original problem (7.1 or 7.2) and its dual.

If any of the variants of the information gathering process discussed above were adopted, the college dean and his staff would do most of the decision making. The department chairmen would serve mostly as technicians or experts who can supply the various coefficients needed to solve the problem. Almost any workable scheme would need to rely heavily upon the knowledge of department chairmen and faculty members, but some schemes could also allow them to participate a bit more in the decision making process.

Kornai and Lipták (29) have described a procedure which could be used to obtain the optimum values for the third

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[1]As before, if no such columns or rows are found the process terminates. If some are found they are used to form the N + 1th version of the problem.

set of operational control instruments ($\rho$ and u) discussed above. As would be expected the vector $\rho$ is set equal to c. The vector of quotas assigned to the various departments, would if their procedure were adopted, be actively used in the attempt to find an optimum vector of quotas $u^*$.

At the outset the college dean would inform each department chairmen of the values taken by those elements of c which correspond to activities supervised by his department. He would then select quotas $u_1, \ldots u_n$ for each of his departments. Each department chairmen would be asked to determine the marginal value to his department of each commodity subject to quota. The college dean uses the marginal value information which he receives to derive a second of quotas. This process would continue until an optimum set of quotas is obtained.

The iterative procedure discussed above is designed to solve 7.8.

$$(7.8) \quad \max_{u \in U} \varphi(u)$$

$$U = \left\{ u = \begin{bmatrix} u_1 \\ \vdots \\ u_n \end{bmatrix} \middle| \sum_{i=1}^{n} u_i \leqq \bar{b}, \; X_i(u_i) \neq \emptyset, \; i = 1, 2 \ldots n \right\}$$

$$\text{where } \varphi(u) = \sum_{i=1}^{n} \max_{x_i \in X_i(u_i)} c_i' x_i$$

If the procedure described by Kornai and Liptāk were adopted, the college dean would, during the Nth phase of the process, solve 7.9.

$$(7.9) \quad \max_{u \in U} \ (W\langle N-1\rangle)' \, u$$

He would use the solution vector $\hat{u}\langle N\rangle$ to compute $u\langle N\rangle$, by letting

$$u\langle N\rangle = \frac{(N-1)}{N} u\langle N-1\rangle + \frac{1}{N} \hat{u}\langle N\rangle, \text{ and}$$

would inform the ith department chairman of the values of the elements of $u_i\langle N\rangle$ and ask him to solve 7.10.

$$(7.10) \quad \min_{v_i \in V_i} v_i' \begin{bmatrix} u_i\langle N\rangle \\ \underline{b}_i \end{bmatrix} = \min_{v_i = \begin{bmatrix} \bar{v}_i \\ \underline{v}_i \end{bmatrix} \in V_i} \left\{ \bar{v}_i' \, u_i\langle N\rangle + \underline{v}_i' \, \underline{b}_i \right\}$$

$$V_i = \left\{ v_i \mid A_i' \, v_i \geqq c_i, \; v_i \geqq 0 \right\}.$$

The department chairmen use the solution vectors $\hat{v}_i\langle N\rangle = \begin{bmatrix} \hat{\bar{v}}_i\langle N\rangle \\ \hat{\underline{v}}_i\langle N\rangle \end{bmatrix}$ $(i = 1, 2 \ldots n)$ to compute $\hat{z}_i\langle N\rangle$ by letting

$\hat{z}_i\langle N\rangle = \left[\hat{\underline{v}}_i\langle N\rangle\right]' \underline{b}_i$. They would then report the vectors $\hat{\bar{v}}_i\langle N\rangle$ and $\hat{z}_i\langle N\rangle$ to the college dean.

The college dean would use the information to form $\hat{W}\langle N\rangle$ and then $W\langle N\rangle$ by letting

$$\hat{W}\langle N\rangle = \begin{bmatrix} \hat{\bar{v}}_1 \\ \vdots \\ \hat{\bar{v}}_n \end{bmatrix}$$

$$\text{and } W\langle N\rangle = \begin{cases} 0 \text{ if } N = 0 \\ \dfrac{(N-1)}{N} W\langle N-1\rangle + \dfrac{1}{N} \hat{W}\langle N\rangle \text{ if } N>0. \end{cases}$$

The college dean now has the information needed to begin the next phase of this process.

In order to help decide when to terminate the process the dean could calculate during each phase two estimates of $\varphi(u^*)$, $\Phi\langle N\rangle$ and $\varphi\langle N\rangle$. $\Phi\langle N\rangle$ provides an upper estimate of $\varphi(u^*)$ (i.e. $\Phi\langle N\rangle \geq \varphi(u^*)$ ) and $\varphi\langle N\rangle$ provides a lower estimate of $\varphi(u^*)$ (i.e. $\varphi\langle N\rangle \leq \varphi(u^*)$ ). If these two estimates coincide, the college dean would know that he has found an optimum set of quotas. The estimates $\Phi\langle N\rangle$ and $\varphi\langle N\rangle$ could be obtained by letting

$$\Phi\langle N\rangle = \left[W\langle N-1\rangle\right]' \hat{u}\langle N\rangle + \sum_{i=1}^{n} Z_i\langle N-1\rangle$$

$$\text{and } \varphi\langle N\rangle = \left[\hat{W}\langle N\rangle\right]' \left[u\langle N\rangle\right] + \sum_{i=1}^{n} \hat{Z}_i\langle N\rangle$$

where $Z_i\langle N\rangle = \frac{(N-1)}{N}\left[Z_i\langle N-1\rangle\right] + \frac{1}{N}\hat{Z}_i\langle N\rangle$ .

$$\varphi(u^*) = \max_{u = \begin{bmatrix} u_1 \\ \vdots \\ u_n \end{bmatrix} \in U} \left\{\sum_{i=1}^{n} \min f(u_i, v_i)\right\}$$

while $$\varphi\langle M\rangle = \sum_{i=1}^{n} \min_{v_i \in V_i} f(u_i\langle M\rangle, v_i)$$

and $$\Phi\langle N\rangle = \max_{u = \begin{bmatrix} u_1 \\ \vdots \\ u_n \end{bmatrix} \in U} \sum_{i=1}^{n} \left\{f(u_i, v_i\langle N-1\rangle)\right\}$$ where

$$f(u_i,\ v_i) = v_i' \begin{bmatrix} u_i \\ b_i \end{bmatrix}$$

Since $u\langle M\rangle$ necessarily belongs to U and (for i = 1, 2 . . n) $v_i\langle N - 1\rangle$ necessarily belongs to $V_i$, it is clear that $\Phi\langle N\rangle \geqq \varphi(u^*) \geqq \varphi\langle M\rangle$ for all integer values of N greater than one and for all integer values of M greater than zero. Thus whenever $\Phi\langle N\rangle = \varphi\langle N\rangle$ or $\Phi\langle N + 1\rangle = \varphi\langle N\rangle$ the process can be terminated. $u\langle N\rangle = \begin{bmatrix} u_1\langle N\rangle \\ \vdots \\ u_n\langle N\rangle \end{bmatrix}$ then yields an optimum system of quotas. By solving the dual of the Nth (if $\Phi\langle N\rangle = \varphi\langle N\rangle$ ) or N + 1th (if $\Phi\langle N + 1\rangle = \varphi\langle N\rangle$ ) phase version of 7.9, an appropriate set of values can be obtained for the vector $\Pi$ (which is part of the second set of operational control instruments discussed).

Although it has been shown that $\lim_{N\to\infty} \Phi\langle N\rangle = \lim_{N\to\infty} \varphi\langle N\rangle = \varphi(u^*)$,[1] the smallest N for which the termination test is satisfied may be very large. This is the major disadvantage of this procedure.[2]

The procedure has some advantages. The major advantage is that the college dean need not keep track of very much information. He need only remember the restrictions which

[1]See (29, p. 154).

[2]Alternative termination conditions have been suggested by Kornai and Lipták (29) which would tend to insure termination in a finite number of phases. They suggest termination whenever $\Phi\langle N\rangle - \varphi\langle N\rangle \leqq \delta$ or $\Phi\langle N + 1\rangle - \varphi\langle N\rangle \leqq \delta$ where $\delta$ is some preselected positive number.

define the set U and the current values of N, $W\langle N\rangle$, $u\langle N\rangle$, $\xi\langle N\rangle$, $\varphi\langle N\rangle$, and $Z_i\langle N\rangle$ for i = 1, 2 . . n. All other information which he receives can be used to update these values and then be discarded. In some cases 7.9 can be solved by "microprogramming". When this is the case the college dean could appoint a separate manager for each commodity subject to quota. These managers could then, acting independently, determine at each phases the appropriate quotas for their commodities. This sort of "microprogramming" would be feasible if the set U can be defined by a set of restrictions none of which involves more than one commodity. In many cases the requirement that $X_i(u_i) \neq \emptyset$ can not be translated into restrictions of that sort.

It seems reasonable to assume that if the college dean were willing to keep track of more of the information that he received, he could use a procedure that required, at most, a finite number of phases to reach an optimum set of quotas. It is known that, for $u \in U$, $\varphi(u) = \sum_{i=1}^{n} \min_j \left[v_{ij}\right]' \begin{bmatrix} u_i \\ b_i \end{bmatrix}$ where the $v_{ij}$'s are the extreme points of the sets $V_i$. The solutions obtained by each department chairman when he solves his version of 7.10 could just as well be restricted to extreme points. This suggests that during the Nth phase the college dean could solve 7.11 rather than 7.9.

$$(7.11)\quad \max \sum_{i=1}^{n} t_i$$

subject to

$$t_i - \left[\overline{v}_i\langle j\rangle\right]' u_i \leqq \hat{Z}_i\langle j\rangle\ , \quad j = 1, 2 \ldots \tilde{N}$$
$$i = 1, \ldots . n$$

$$\sum_{i=1}^{n} u_i \leqq \overline{b}$$

$$X_i(u_i) \neq \emptyset^{1},\ j = 1, \ldots . n$$

$$t_i,\ u_i \text{ unrestricted}, i = 1, \ldots n$$

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[1]The dual of 7.11 can easily be derived from the dual of 7.2. If this were done it would be apparent that the restrictions $X_i(u_i) \neq \emptyset$ ($i = 1, \ldots n$) have the form

$$-\left[\overline{v}_i\ \langle l\rangle\right]' u_i \leqq Z_i\langle l\rangle\ , l = 1, 2 \ldots K_i,\ i = 1, 2 \ldots n$$

where the $v_i\langle l\rangle$'s $\left( = \begin{bmatrix} \overline{v}_i\ \langle l\rangle \\ \underline{v}_i\ \langle l\rangle \end{bmatrix}\right)$ are the extreme points (if any) of the sets $V_i(0) = \left\{ v_i \mid A_i'\ v_i \geqq 0,\ 1'v_i = 1,\ v_i \geqq 0 \right\}$. This means that the restrictions needed to guarantee $X_i(u_i) \neq \emptyset$ (for $i = 1, \ldots n$) could be generated as the process went along. If this route were taken the problems 7.12 would be solved as usual during each phase unless an unbounded solution was obtained. If an unbounded solution was obtained (for department $\tilde{i}$ and phase $\tilde{N}$) the problem would be solved again after substituting $V_i(0)$ for $V_i$. The rest of the procedure is the same as that described except that (a) the $Z_{\tilde{i}}\langle\tilde{N}\rangle$'s and $\overline{v}_{\tilde{i}}\langle\tilde{N}\rangle$'s resulting from minimization on $V_{\tilde{i}}(0)$ are always reported to the college dean; and (b) the fact that they are based on homogeneous solutions is also reported so that the college dean will know how to use the information.

The $\hat{t}_i\langle N\rangle$ 's which along with the $\hat{u}_i\langle N\rangle$ 's solve the Nth phase version of 7.11 provide the college dean with an estimate of the optimum values of the departmental objective functions (solutions to 7.12) which would result if the quotas $\hat{u}_i\langle N\rangle$ were selected.

The college dean would report the vectors $\hat{u}_i\langle N\rangle$ and $\hat{t}_i\langle N\rangle$ (for i = 1, 2 . . n) to the department chairmen and ask them to solve 7.12.

$$(7.12)\quad \min_{v_i \in V_i} v_i' \begin{bmatrix} \hat{u}_i\langle N\rangle \\ \underline{b}_i \end{bmatrix} = \hat{\Omega}_i\langle N\rangle$$

$$V_i = \left\{ v_i \mid A_i' v_i \geqq c_i,\ v_i \geqq 0 \right\}$$

They would calculate the vectors $\hat{Z}_i\langle N\rangle$ by letting $\hat{Z}_i\langle N\rangle = \left[\underline{\hat{v}}_i\langle N\rangle\right]' \underline{b}_i$ and then report the $\hat{\underline{v}}_i\langle N\rangle$ 's and $\hat{Z}_i\langle N\rangle$ 's to the college dean for those departments (those i's) for which $\hat{\Omega}_i\langle N\rangle < \hat{t}_i\langle N\rangle$ (that is, for those departments for which the college dean overestimated the value of the departmental objective function).

This procedure would continue until at the end of some phase no $\hat{\Omega}_i\langle N\rangle$ is less than $\hat{t}_i\langle N\rangle$ .[1] If the phase for which this occurs is phase M, then an optimum set of quotas is given by $\hat{u}\langle M\rangle$ . An appropriate set of values for the elements of $\Pi$ could be obtained by solving the dual of the

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[1]The termination could remain the same as before if $\Phi\langle N\rangle$ is redefined to be equal to $\sum_{i=1}^{n} \hat{t}_i\langle N\rangle$ .

Mth phase version of the dual of 7.11.

The number of phases required will be finite since there are at most a finite number of extreme points for each of the sets $V_i$. The maximum number of phases required is equal to $m_i$ where $m_i$[1] is the number of extreme points of $V_i$.

The last procedure for discovering appropriate values for the dean's control instruments to be discussed involves the use of prices rather than quotas to guide the information gathering process. If the college dean were to adopt this procedure he would, during each phase, establish prices on those commodities important to more than one department and ask the department chairmen to report the quotas which they would request at those prices. He would use this set of quotas to generate a new set of prices which would be used to generate a new set of quotas, and so forth. This procedure must be regarded as being more passive than the ones described above since the college dean does not, under this procedure, actively set or manipulate the quotas.

This last procedure is a straightforward application of the decomposition principle. During the first stage the college dean uses the information received from the department

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[1] $m_i$ equals the number of extreme points of $V_i$ plus the number of extreme points of $V_i(0)$ if the modification suggested in the preceding footnote is adopted.

chairman to determine a feasible set of quotas. If he is successful, he then proceeds to the second stage. In the second stage he attempts to find an optimum set of quotas.

Before initiating the procedure, the dean insures that each department is aware of the relevant components of c. During the Nth phase of the first stage the college dean first solves the dual of the Nth version of 7.13.

$$(7.13) \quad \max_{r \in R} - 1' r$$

$$R\langle N\rangle = \left\{ r = \begin{bmatrix} \bar{r} \\ \underline{r}_1 \\ \vdots \\ \underline{r}_n \end{bmatrix} \middle| \begin{array}{l} \sum_{i=1}^{n} \sum_{j=1}^{N-1} \left[u_i\langle j\rangle\right] \left[\lambda_i\langle j\rangle\right] - \bar{r} \leqq \bar{b} \\ \sum_{j=1}^{N-1} \lambda_i\langle j\rangle + \underline{r}_i = 1, \; i = 1, 2 \ldots n \\ r \geqq 0, \; \lambda_i\langle j\rangle \geqq 0, \; \begin{array}{l} j = 1, \ldots N-1 \\ i = 1, \ldots\ldots n \end{array} \end{array} \right\}^{1}$$

After obtaining vector $\hat{v}\langle N\rangle = \begin{bmatrix} \hat{\bar{v}}\langle N\rangle \\ \hat{q}_1\langle N\rangle \\ \vdots \\ \hat{q}_n\langle N\rangle \end{bmatrix}$ he reports its components to the department chairmen and asks them to solve problems 7.14.

$$(7.14) \quad \hat{\Omega}_i\langle N\rangle = \max_{x_i \in X_i} \left\{ - \left[\hat{\bar{v}}\langle N\rangle\right]' \bar{A}_i \, x_i - \hat{q}_i\langle N\rangle \right\}$$

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[1]For N = 1 R reduces to

$$R\langle 1\rangle = \left\{ r = \begin{bmatrix} \bar{r} \\ \underline{r}_1 \\ \underline{r}_n \end{bmatrix} \middle| \begin{array}{l} - \bar{r} \leqq \bar{b} \\ \underline{r}_i = 1, \; i = 1, \ldots n \\ r \geqq 0 \end{array} \right\}$$

After obtaining the solution to his version of 7.14 each department chairman would check to see if $\hat{\Omega}_i\langle N\rangle$ is greater than zero. If not he merely reports the fact that it is not to the college dean. If it is greater than zero he computes the vector of quotas which is consistent with his solution, $\hat{x}_i\langle N\rangle$ to 7.14 by setting $u_i\langle N\rangle = \overline{A}_i\hat{x}_i\langle N\rangle$. He also computes $\gamma_i\langle N\rangle$ by setting $\gamma_i\langle N\rangle = c_i' \hat{x}_i\langle N\rangle$. He then reports $\gamma_i\langle N\rangle$ and $u_i\langle N\rangle$ to the college dean. The college dean uses these elements to set up the (N + 1)th phase version of 7.13.

Let $N_1$ be the smallest integer for which $\max_{r\in R\langle N_1\rangle} [-1'r]$ is equal to zero. (This implies that a feasible set of quotas has been found.) The ($N_1$ - 1)th phase will then have been the last full phase of stage 1. The $N_1$th phase consists of solving the $N_1$th phase version of 7.13, discarding the r vector (or fixing its elements equal to zero), and completing the $N_1$th phase of the second stage.

During the Nth phase of the second stage the college dean would solve the dual of 7.15.

(7.15)
$$\max \sum_{i=1}^{n} \sum_{j=1}^{N-1} [\gamma_i\langle j\rangle] [\lambda_i\langle j\rangle]$$

subject to

$$\sum_{i=1}^{n} \sum_{j=1}^{N-1} [u_i\langle j\rangle] [\lambda_i\langle j\rangle] \leqq \overline{b}$$

$$\sum_{j=1}^{n-1} \lambda_i\langle j\rangle = 1, \quad i = 1,2 \,.\,.\, n$$

$$\lambda_i\langle j\rangle \geqq 0 \quad , \quad j = 1,2 \,.\,.\, N-1, \quad i = 1,2 \,.\,.\,.\,.\, n$$

He reports the elements of the solution vector $\hat{\bar{v}}\langle N\rangle$ to the department chairmen, and asks them to solve problems 7.16.

$$(7.16)\quad \hat{\Omega}_i\langle N\rangle = \max_{x_i \in \underline{X}_i} \left\{ \left(c_i' - \left[\hat{\bar{v}}\langle N\rangle\right]' \bar{A}_i\right) x_i - \hat{q}_i\langle N\rangle \right\}^{1}$$

The ith department chairman then reports the values of $\gamma_i\langle N\rangle$ and $u_i\langle N\rangle$ to the college dean if $\hat{\Omega}_i\langle N\rangle$ is greater than zero. $\gamma_i\langle N\rangle$ and $u_i\langle N\rangle$ can be obtained by letting

$\gamma_i\langle N\rangle = c_i' \; \hat{x}_i\langle N\rangle$ and

$u_i\langle N\rangle = \bar{A}_i \; \hat{x}_i\langle N\rangle$ where $\hat{x}_i\langle N\rangle$ is any extreme point solution to his version of 7.16.

The second stage terminates at the end of the first phase, say the $N_2$th, for which $\hat{\Omega}_i\langle N_2\rangle = 0$ for $i = 1,2 \ldots n$. $N_2$ will be finite since each of the sets $\underline{X}_i$ (and $\underline{X}_i(0)$ ) has only a finite number of extreme points.

An optimum set of prices for those commodities affecting more than one department can be obtained directly from the solution to the dual of the $N_2$th phase version of 7.15. An optimum set of quotas can be obtained by setting

$$u_i^* = \sum_{j=1}^{N_2-1} \left[\lambda_i^*\langle j\rangle\right]\left[u_i\langle j\rangle\right] + \sum_{l=1}^{N_2-1} \left[\mu_i^*\langle l\rangle\right]\left[u_i\langle l\rangle\right]$$

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[1]In the event that an unbounded solution was obtained to 7.14 or 7.16 the procedure outlined in Chapter 4 could be used. The college dean would then be warned that the solution to the original (7.14 or 7.16) problem was unbounded (for some department) and that (for that department) the solution reported was obtained by solving the modified version of 7.14 or 7.16.

where the $\lambda_i^*\langle j\rangle$ 's and $\mu_i^*\langle l\rangle$'s solve the $N_2$th phase version of 7.15.

Several sets of operational control instruments and several procedures for obtaining appropriate values for these instruments have been discussed. Each of the procedures for obtaining values for these instruments could be used to obtain appropriate values for the instruments of more than one of the sets of operational control instruments. Table 2 outlines some of the possible combinations of operational control instruments and procedures for obtaining appropriate values for them. The three cells labeled "not considered" represent schemes which would be feasible if the information gathering procedures discussed were modified somewhat. They will not be considered here because they require a largely unnecessary flow of information. Two of them require first that the department chairmen compute the optimum activity levels, second that they send this information to the college dean, and third that the college dean ask each department chairman to implement exactly those activity levels which the department chairman has just computed.

Instead of solving 7.1 or 7.2 directly it should be possible to obtain a solution to 7.2 (and thus to 7.1) in a way which would allow simulation of one of the schemes in Table 2.

Schemes I, II, and III are inappropriate for this purpose primarily because the simulatable portions of these

Table 2. Alternative decision making schemes.

| Types of operational control | Major instruments used / Major instrments used | Alternative methods for obtaining optimum values of operational control instruments: Centralized methods | Decentralized methods | | |
|---|---|---|---|---|---|
| | | Activity levels, prices, or both | Quotas (only new information used) | Quotas (all relevant information used) | Prices |
| Centralized | Activity levels | Scheme I | not considered | not considered | not considered |
| Decentralized | Prices | Scheme II | Scheme IV.a | Scheme IV.b | Scheme VI |
| | Quotas | Scheme III | Scheme V.a | Scheme V.b | Scheme VII |

schemes are too direct. The major difference between these schemes and direct solution is that between each iteration new columns or rows (activities or constraints) are added to the model. The difficult part of this is not adding the column or row but conducting the research on which to base the new column or row.

Schemes II, IV.a, IV.b, and VI involve the use of a set of operational control instruments which may not be effective. Therefore, they will not be adopted. Instead the control instruments which they would use will be tested for the particular model which has been formulated after a solution has been arrived at by another method.

Schemes IV.a and V.a will not be used primarily because of the possibility of having to choose either an approximate solution or a long (large number of phases) solution procedure.

This leaves schemes V.b and VII. Either of these schemes could be used to obtain a solution to version two of the college model. However, scheme VII seems to correspond more closely to the actual decision making processes of many academic institutions. That is, as part of the decision making process department chairmen are often asked to compute and submit budget "requirements" on the basis of "guidelines" which the college dean had previously issued. If the requirements exceed the total amount of funds which the college dean has available, the college dean may ask that alternative sets of "requirements" be prepared which require less funds (i.e. which place a higher opportunity cost on budget resources). Somehow at the end of this process "firm" budgets are obtained. The department chairmen then operate their departments as best they can with the resources which they have been allocated.

## VIII. THE COLLEGE MODEL: SOLUTION AND EXTENSIONS

When applied to the college model, 7.8 can be written

$$\max \left\{ \max_{x_a \in X_a(u_a)} c_a' x_a + \max_{x_b \in X_b(u_b)} c_b' x_b + \max_{x_d \in X_d(u_d)} c_d' x_d + \max_{x_e \in X_e(u_e)} c_e' x_e \right\}$$

subject to

$$u_a + u_b + u_d + u_e \leqq \bar{b}$$

$$X_i(u_i) \neq \emptyset \text{ for } i = a, b, d, \text{ and } e.$$

The first set of constraints can be rewritten (taking advantage of the fact that the best values of some elements are known to be zero) as

| | | | | | |
|---|---|---|---|---|---|
| (8.1) | $u_{a1}$ | | | $+ u_{e1}$ | $\leqq -725$ |
| (8.2) | | $u_{b2}$ | | $+ u_{e2}$ | $\leqq -400$ |
| (8.3) | $u_{a3}$ | $+ u_{b3}$ | | | $\leqq -160$ |
| (8.4) | $u_{a4}$ | | | $+ u_{e4}$ | $\leqq -0.75$ |
| (8.5) | | $u_{b5}$ | | $+ u_{e5}$ | $\leqq -1.25$ |
| (8.6) | $u_{a6}$ | $+ u_{b6}$ | | | $\leqq -2.75$ |
| (8.7) | $u_{a7}$ | $+ u_{b7}$ | $+ u_{d7}$ | $+ u_{e7}$ | $\leqq 0$ |
| (8.8) | $u_{a8}$ | | $+ u_{d8}$ | $+ u_{e8}$ | $\leqq 27$ |
| (8.9) | $u_{a9}$ | $+ u_{b9}$ | | $+ u_{e9}$ | $\leqq 19{,}125$ |
| (8.10) | $u_{a10}$ | $+ u_{b10}$ | $+ u_{d10}$ | $+ u_{e10}$ | $\leqq 1{,}095{,}000.$ |

The first three constraints (8.1, 8.2, and 8.3) deal with instruction produced by some departments of the college for graduate students in other departments. The first type of instruction (constraint 8.1) considered is instruction supplied by department A for Economics graduate students and for graduate students of other colleges. The second type of instruction (constraint 8.2) is supplied by department B to Economics graduate students and graduate students of other colleges. The third type of instruction (constraint 8.3) is supplied by department B to graduate students of department A and to graduate students of other colleges.

The next three constraints deal with teaching services supplied by departments of the college for use by other departments. In this model these transfers account for the fact that the supervisory committees which guide the study of graduate students often include one or more members from the departments in which the student takes supporting courses. The pattern of production and use for the three types of teaching services considered is the same as for the three types of instruction considered above.

The next constraint (constraint 8.7) insures that the rate of small classroom use does not exceed the rate of classroom allocation provided by "department" D. Constraint 8.8 insures that the rate of large classroom use does not exceed the allocation granted by the university president. (27 units per year).

Constraint 8.9 insures that the amount of office space used by the college does not exceed that allocated to the college by the university president. Constraint 8.10

insures that the rate of use of teaching budget funds does not exceed the rate authorized by the university president.

An optimal u vector was computed using the passive (based on the decomposition lemma) procedure discussed in Chapter 7. In most cases the college dean would not actually need to use the first stage of that procedure because he would be able to select a feasible set of quotas at the outset. However, for this example the computations were were carried out almost exactly as suggested in Chapter 7.

Department models A, B, and D had (for some prices) unbounded solutions. These were treated essentially as suggested in Chapter 4.[1] The model of department A also had (for some prices) solutions that were nearly unbounded. In order to keep these solutions manageable the set $\underline{X}_a$ was bounded by requiring that $u_{a1} \geqq y_{a1} \geqq - 5000$. The particular bound chosen was a good deal smaller (larger in absolute value) than it needed to be. Most of the components of u could have been bounded on the basis of the sort of *a priori* information which the various decision makers would ordinarily possess. In some cases, however, adding these bound would have resulted in no gain and may have increased the number of phases required to obtain a solution.

The prices which the college dean would have transmitted

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[1]The only difference was that the restriction $1'x_i = 1.02$ was used in some cases instead of the restriction $1'x_i = 1$.

to the departments during each phase of the solution procedure are shown in Table 40. The quotas which the department chairmen would have requested at these prices are shown in Tables 41, 42, 43, and 44.

The sixth phase was the first phase of stage two. The total number of phases required was 22.

An optimum set of quotas is shown in Table 45. The solutions to the departmental models are, for this set of quotas, shown in Tables 14, 27, 50, and 64. The dual solutions are presented in Tables 15, 28, 51, and 65.

The final (phase 22) version of 7.15 can be used to examine the effectiveness of several alternative sets of control instruments.

The ineffectiveness of prices alone as instruments to control the production and use of those commodities affecting more than one department is apparent. The "best" prices would be part of the vector which solves the dual of the final version of 7.15. The optimal basis for the final version of 7.15 includes two vectors which were based on homogeneous solutions. It also includes 4 vectors based on extreme points of $\underline{X}_A$, 3 vectors based on extreme points of $\underline{X}_B$, and 3 vectors based on extreme points of $\underline{X}_E$. If prices were used as control instruments any basic vector which was based on an extreme point would be, for some department, an optimal vector of commodity use rates. As a result, it is unlikely that the use rates that the departments would pick (if guided only by prices) would result in a solution to the college model which would be both feasible and optimal.

The particular structure of the model considered here suggests that a "mixed" set of control instruments might be more effective. If this set of control instruments were adopted, those departments which provide instruction (and teaching services) used by other departments in the college would be required to meet or exceed certain output quotas. Quotas would also be used to control the use of classrooms, office space and teaching budget funds. Prices would be used to control the use of instruction (and teaching services) produced by other departments in the college.

It is apparent that this set of instruments would effectively control resource use (and production) by departments B and D since these departments do not use any inputs produced by other departments within the college.

This set of control instruments can not be judged ineffective merely by examining the final (phase 22) version of 7.15. For each department the number of basic activities is less than the number of constraints that would be imposed by the set of control instruments being considered. Thus the set of control instruments might be effective.

The effectiveness of this set of control instruments was tested by first solving the two linear programs (for departments A and E) implied by the set of instruments and then testing the sensitivity of the solutions to changes in the prices assigned to instruction and teaching services furnished by other departments. In both cases the set of control instruments was found effective.

Is is customary for college deans to be actively involved in the allocation of funds and space but ordinarily they do not become as involved in the allocation of instruction and teaching services. The set of instruments considered above is of interest primarily because it suggests that the college dean need not be actively involved in the allocation of instruction and teaching services.

The success of the test made above should not be interpreted as a confirmation of this hypothesis. It merely indicates that for the model used here there exist equilibrium prices for, and quantities of, instruction and teaching services. It does not show that this equilibrium is in any sense a stable equilibrium. That is, it does not demonstrate that the equilibrium quantities and prices could be determined without the college dean (or some other person) being actively involved in the process.[1]

[1]Of course deans don't usually get involved in this. More commonly departments predict the "demand" for the service instruction and service resources which they provide and plan on the basis of this prediction. Viewed within the framework of the model considered here it is apparent that such an approach would ordinarily give an infeasible or suboptimal solution. However, in a model which allowed for product improvement or worsening (and thus for more flexible class sizes and so forth) this treatment of service instruction and service resources would undoubtedly fare much better. In either event this method of handling this matter probably reduce the number of phases required to obtain an optimum set of quotas for the other commodities.

The treatment of the allocation of classroom use in the model was quite arbitrary. At least two alternatives could be suggested. One would involve requiring that the rate of classroom use not exceed some fixed level. Unless this rate is fairly large relative to the number of classrooms available (that is, unless it permits a high use rate per classroom) this alternative would seem to be even more arbitrary and unrealistic than the approach actually taken.

Another alternative would be to treat the services of classrooms as free goods. This alternative was implemented by eliminating the constraints on classroom use. The total college budget was decreased by the amount formerly allocated to the classroom allocation "department" in order to isolate the "price effect" of this change from the "budget effect".

As would be expected, the value of the college dean's objective function increased. An exact solution was computed, and it was determined that the increase in the value of the college dean's objective function would be relatively small. Table 47 presents an optimal solution to the dual of 7.15. Table 48 presents an optimal set of quotas.

The models presented in this study have treated the rate of undergraduate degree production as being a variable which the various decision makers could not, or were reluctant to, actively control. While it was felt that this is not exactly the case in many educational institutions the loss in reality was probably more than offset by the gain in ease of formulation afforded by this treatment.

Treating the rate of undergraduate degree production as a controllable variable would effectively increase the number of commodities whose use and production rates must be coordinated by the college dean (or by some other decision maker). Graduate students in any given discipline may take the bulk of their courses in 2 to 4 departments but it is not uncommon for undergraduate students majoring in any given discipline to take courses in as many as fifteen different departments.

The college model presented in Chapter 7 involved a college having three academic departments. Most colleges have more departments. For such a college the number of commodities requiring coordination by the college dean (or by some other decision maker) would be greater than for a college having only three departments.

The decision making schemes discussed in Chapter 7 may be extended to multi-level decision making schemes. In fact it is possible to regard the schemes discussed there as being the final parts of longer multi-level decision making schemes.

In the case of the college considered earlier it is reasonable to assume that some of the commodities relevant to a model of that college may also be relevant to at least one other college within the university. The university president could thus be involved in the allocation of these commodities among the various colleges in the university.

If the words "university president" were substituted for the words "college dean" and the words "college deans" were substituted for the words "department chairmen" the schemes discussed in Chapter 7 could be used to arrive at an allocation of these commodities between the colleges. After the university president has made these allocations, the decision making process could continue in each college as originally described in Chapter 7.

In the college model presented earlier the college dean's objective function involved more than twenty flexible targets. If the decision making schemes discussed were to be extended to the management of all of the state controlled educational institutions in a state or to the management of all state controlled institutions it is apparent that these schemes could require that some decision maker's objective function involve a very large number of flexible targets. It seems unreasonable to believe that such a decision maker would feel qualified to assign weights to each of these targets. Fortunately the requirement that he be able to do this may be relaxed a bit.

It may be that such a decision maker might adopt a view of the outputs of educational institutions which permits him to consider a smaller number of outputs. There are many different ways in which he could view the outputs of the state's educational institutions. For example, he might not be concerned with the numbers of degrees of various types awarded or with the number of pages published by faculty members except as these outputs have their impacts on the state's supply of trained manpower, on the economic development of the state, or on some other measure of the state's performance or potential.

If this decision maker can communicate the weights which he would assign to each of these more final outputs clearly enough, it may be possible for each college dean or department chairman to translate them into weights for the outputs of a college or department.

The decision making schemes considered in Chapter 7 dealt only with linear objective functions. In general, these schemes cannot cope with non-linear objective functions. However, some sorts of non-linear objective functions can be handled without too much difficulty. One class of such functions includes functions having the form of the so-called "utility tree".[1] An objective function having this form can be written as

$$O = F\left[f_1(x_1),\ f_2(x_2),\ .\ .\ .\ f_n(x_n)\right].$$

[1]Strotz (39) discusses some of the implications of this type of utility function.

If F is a concave in both the $f_i$'s and the $x_i$'s and if each $f_j$ is concave in the activity levels $x_j$ then some of the schemes discussed in Chapter 7 could, with some modification, be applied.

If the assumptions made above hold, the set of control instruments which involves the use of quotas to control the use and production of commodities affecting more than one department would still be an effective set of instruments. That is, if the ith department chairman maximized $f_i(x_i)$ on the set $X_i(u_i)$ there would exist vectors $u_i^*$ such that $x_i^*(u_i^*)$ (for $i = 1, 2 . . . , n$) would maximize F on the set X. Much the same sort of construction proof that was used earlier for the linear case could be used to demonstrate the validity of this assertion.

The passive approach to the determination of an optimal set of quotas as discussed for the linear case could, in principle, be modified to give an optimal set of quotas for this particular non-linear case. Stage one of this approach would proceed exactly as described in Chapter 7. During the Nth phase of the second stage of this procedure the college dean would solve the dual of 8.11 rather than the dual of 7.15.

$$(8.11) \quad \max F(Z_1, \ldots Z_n)$$

$$\text{subject to } Z_i = \sum_{j=1}^{N-1} \left[\lambda_i\langle j\rangle\right] f_i(\hat{x}_i\langle j\rangle)\ , \quad i = 1, 2 \ldots n$$

$$\sum_{i=1}^{n} \sum_{j=1}^{N-1} \left[u_i\langle j\rangle\right]\left[\lambda_i\langle j\rangle\right] \leqq \bar{b}$$

$$\sum_{j=1}^{N-1} \left[\lambda_i\langle j\rangle\right] = 1, \ i = 1, 2 \ldots n$$

$$\lambda_i\langle j\rangle \geqq 0 \quad j = 1, 2 \ldots N - 1$$
$$i = 1, 2 \ldots n$$

(If F is non-linear the college dean may have to solve 8.11 before its dual can be solved.) The college dean would report the elements of the dual solution vector

$$\hat{v}\langle N\rangle = \begin{bmatrix} \hat{\alpha}_1\langle N\rangle \\ \vdots \\ \hat{\alpha}_n\langle N\rangle \\ \hat{\bar{v}}\langle N\rangle \\ \hat{q}_1\langle N\rangle \\ \vdots \\ \hat{q}_n\langle N\rangle \end{bmatrix}$$

to the department chairmen. The ith department chairmen would solve (during the Nth phase) problem

$$(8.12) \quad \hat{\Omega}_i\langle N\rangle = \max_{x_i \in X_i} \left\{ \left[\hat{\alpha}_i\langle N\rangle\right] f_i(x_i) - \left[\hat{\bar{v}}\langle N\rangle\right]' \bar{A}_i \ x_i \right\}$$

If $\hat{\Omega}_i\langle N\rangle$ is greater than $\hat{q}_i\langle N\rangle$ he computes $f_i(\hat{x}_i\langle N\rangle)$ and (as before) $u_i\langle N\rangle$, and reports them to the college dean. The college dean uses these elements to form the

N + 1th phase version of 8.11.

There are at least two ways in which adopting a non-linear objective function of the type described above could lead to complications. First, if the functions F and $f, . . f_n$ are nonlinear (concave) the college dean and department chairmen may encounter somewhat more difficulty when attempting to solve 8.11 and 8.12. Secondly, if the $f_i$'s are nonlinear (concave) the solutions to problems 8.12 will not always be extreme points of the sets $\underline{X}_i$. Although the procedure would still converge monotonically to a maximum of F on the set X, the number of phases required to obtain an exact solution to the problem as formulated might not be finite. This drawback is probably not too severe, however, since an exact solution may not always be required.

If the $f_i$'s are linear the solutions to the problems 8.12 can be limited to the extreme points of $\underline{X}_i$ (or of $\underline{X}_i(0)$) and solution in a finite number of phases would be assured.

It has been assumed that preference functions of the department chairmen were consistent with that of the college dean. Each target which was fixed from the point of view of the college dean was, if it affected only one department, assumed to be regarded as a fixed target by one of the department chairmen. The flexible targets were given the same relative weights by both the college dean and the department chairmen.

If the preference functions of the department chairmen are not consistent with that of the college dean much of the discussion of Chapter 7 is no longer relevant. Although they may not regard them as targets but merely as restrictions, the department chairmen would undoubtedly still insure that the fixed targets and quotas set by the college dean are met. However, the college dean might have a difficult time maximizing his objective function if he attempts to use a decentralized decision making scheme.

In order to try to achieve a maximum value of his objective function the college dean may impose restrictions in addition to those needed to guarantee that fixed targets are met. He may impose additional restrictions on activity levels, on output levels, on input levels, or on input-output combinations.

If the college dean does not impose these additional restrictions but chooses instead to do the best that he can using only quotas, he may have a more difficult time finding a best set of quotas. If the department chairmen's objective functions were consistent with his own, the college dean could be assured that the vector $x_i^*(u_i)$ chosen by the ith department chairman would be such that $c_i' \; x_i^*(u_i)$ would not decrease if some elements of $u_i$ were increased without lowering any of the other elements. He could also be assured that $c_i' \; x_i^*(u_i)$ is concave in $u_i$. However if the ith department chairman is seeking to maximize $d_i' \; x_i^*(u_i)$ $(d_i \neq c_i)$ $c_i' \; x_i^*(u_i)$ need not be a concave nondecreasing function of $u_i$.

The college dean could allocate more resources to department i and get less in return (as measured by $c_i' x_i^*(u_i)$ ).

In this sort of situation the college dean would often, when deciding whether or not to allocate additional resources to any department, need to ask the department chairman what he would do with these additional resources. Even if the department chairmen are not using the same set of prices for their planning, dissemination of the college dean's prices may still be helpful since the department chairmen would then know what the costs (in terms of the changes in their output plans that would be required to obtain more resources) of additional resources are. By announcing his prices the college dean may tend to cause the department chairmen to act as if they were maximizing objective functions consistent with that of the college dean.[1]

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[1]This convergence of objective function weights need not be one-sided. If the college dean's prices (or weights) are announced, the department chairmen may convince him that his weights are wrong. That is, they may be able to convince him that his evaluations of some of the outputs are incorrect.

# IX. CONCLUSION

Several models have been presented and on the basis of these models several decision making schemes have been discussed. Both the models and the decision making schemes would have to be modified if they were to fit any particular institution. Some of the deficiencies may be quite general. That is, there may be features common to nearly all educational institutions which have not been treated adequately. For any particular institution it is certain that there are features which have been treated inadequately.

Some potential deficiencies in the models have been pointed out in Chapters 6 and 7. These will not be repeated here.

The deficiencies and potential limitations of the decision making schemes have not been discussed quite as completely. At least two of these limitations are particularly obvious. The first is that the schemes discussed placed all of the decision making responsibility on the college dean and department chairmen, whereas many other persons are also involved in the decision making process.

A second limitation is the large number of phases apparently required to reach a final decision. This is more serious than the treatment here would indicate. For the purposes of the results obtained in Chapter 8, it was assumed that the model had been completely specified at the beginning of the decision process. In practice, however, the department chairmen would tend to formulate their models less completely initially and then improve upon them as necessary. Thus a good deal more work may be required during each phase than is apparent from Chapters 7 and 8.

The large number of phases required to reach a final decision is due in part to the particular formulation chosen. If the model or the decision making scheme were modified so that the college dean were actively involved in the allocation of fewer resources then the number of phases required would tend to decrease.

The department chairmen may ordinarily be able to supply the college dean with a feasible set of quota requests at the beginning of the decision process. If this were done the phases that would otherwise be needed to obtain a feasible solution could be eliminated. In practice several other phases would probably be eliminated since the college dean would not be expected to continue the process for as many phases as would be required to obtain an exact solution. As can be seen from Table 46 the improvements in the value of the objective function were all rather small after about the 13th or 14th phase.

The thesis has dealt with models which can be solved for optimum output and activity levels and with decision making schemes which promise to yield optimum solutions to these models. However, the major advantage to be derived from the use of models may turn out to be the organized ways they provide for looking at particular problems rather than the numerical solutions that can be obtained.

The use of a model forces the decision maker to set forth the major variables and the most important relationships which are involved in a particular problem. Having set forth the elements of the situation as he sees it, he may be in a better position to determine what additional information is needed and what existing information may be ignored or at least set aside for a while. Once he has outlined the general framework of the problem he may be able to decentralize the task of filling in the exact details for each of its sectors or subsectors.

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# XII. APPENDIX A

Table 3. Final products, department A.

| Commodity number | Product description | Unit of measurement |
|---|---|---|
| $a_0$[a] | Ph.D. degrees awarded by department A | One degree per academic (9-10 month) year. |
| $a_{00}$[a] | Masters degrees awarded by department A | One degree per academic (9-10 month) year. |
| $a_1$ | Training of undergraduate students majoring in discipline A. | One undergraduate unit per year. One unit is amount of instruction (and departmental administration) in discipline A which is required to produce one Bachelor's degree (in discipline A) per year. |
| $a_2$ | Instruction in the principles of discipline A. | One instructional unit per year. An instructional unit is one (3 quarter hours) course taught to one student. |
| $a_3$ | Advanced undergraduate (and minor graduate) instruction in discipline A. | Same as above. |
| $a_4$ | Graduate level instruction in the methods of discipline A. | Same as above. |
| $a_5$ | Graduate instruction in the theory of discipline A. | Same as above. |

[a] Commodities $a_0$ and $a_{00}$ are commodities for which no constraints have been included in the model.

Table 3. (continued)

| Commodity number | Product description | Unit of measurement |
|---|---|---|
| $a_6$ | Faculty teaching services (advanced undergraduate or minor graduate teaching). | One unit of teaching per year. A unit of teaching is the amount required to teach (including preparation) one (3 quarter hours) course. |
| $a_7$ | Faculty teaching services (graduate level "methods" teaching). | Same as above. |
| $a_8$ | Publications by "theory" faculty members. | One publication (20 pages of publications) per year. |
| $a_9$ | Publications by "methods" faculty members. | Same as above. |
| $a_{10}$ | Publications by "applications" faculty members. | Same as above. |

Table 4. Intermediate products, department A.

| Commodity number | Product description | Unit of measurement |
|---|---|---|
| $a_{11}$ | Faculty teaching services (graduate level "theory" teaching). | One unit of teaching per year. A unit of teaching is the amount required to teach (including preparation) one (3 quarter hours) course. |
| $a_{12}$ | Faculty teaching services (under-graduate principle teaching). | Same as above. |
| $a_{13}$ | Teaching services provided by graduate teaching assistants. | One unit of teaching assistance per year. One unit is the amount of assistance required to "teach" one recitation section or one hour of laboratory per week for one quarter. |
| $a_{14}$ | Teaching services provided by graduate student instructors. | Same as for faculty instruc-tional services. |
| $a_{15}$ | Secretarial and clerical assistance used by teaching activities. | One woman year (9-10 months) per year. |
| $a_{16}$ | Research assistance ("theory" research) provided by Masters degree candidates. | One half (4.5 months full time equivalent) man year per year. |
| $a_{17}$ | Research assistance ("methods" research) provided by Masters degree candidates. | Same as above. |

Table 4. (continued).

| Commodity number | Product description | Unit of measurement |
|---|---|---|
| $a_{18}$ | Research assistance ("applications" research) provided by Masters degree candidates. | One half (4.5 months full time equivalent) man year per year. |
| $a_{19}$ | Research assistance ("theory" research provided by Ph.D. candidates. | Same as above. |
| $a_{20}$ | Research assistance ("methods" research provided by Ph.D. candidates. | Same as above. |
| $a_{21}$ | Research assistance ("applications" research) provided by Ph.D. candidates. | Same as above. |
| $a_{22}$ | Secretarial and clerical assistance used by "theory" research activities. | One woman year (9-10 months) per year. |
| $a_{23}$ | Secretarial and clerical assistance used by "methods" research activities. | Same as above. |
| $a_{24}$ | Secretarial and clerical assistance used by "applications" research activities. | Same as above. |
| $a_{25}$ | Research services provided by "theory" faculty members. | One man year (9-10 months) per year. |
| $a_{26}$ | Research services provided by "methods" faculty members. | Same as above. |
| $a_{27}$ | Research services provided by "applications" faculty members. | Same as above. |

Table 4. (continued).

| Commodity number | Product description | Unit of measurement |
|---|---|---|
| $a_{28}$ | Counselling and advising of undergraduate students. | One counselling unit per year. One unit is the amount of advising required per student (discipline A major) per quarter. |
| $a_{29}$ | Counselling and advising of graduate students. | Same as above. |
| $a_{30}$ | Thesis (and related) supervision of graduate students. | The supervision of 3 thesis credit hours per year. |

Table 5. Primary products, department A.

| Commodity number | Product description | Unit of measurement |
|---|---|---|
| $a_{31}$ | Instruction provided for department A's graduate students by departments outside of A's college. | Same as for $a_2$. |
| $a_{32}$ | Teaching services provided to department A by departments outside of A's college. | Same as for $a_6$. |
| $a_{33}$ | Instruction provided for department A's graduate students by department B. | Same as for $a_2$. |
| $a_{34}$ | Teaching services provided to department A by department B's faculty members. | Same as for $a_6$. |
| $a_{35}$ | Large classroom use. | One classroom unit per year. One unit is the amount (no. of hours) of classroom use required to conduct a class meeting one hour per week for one quarter. |
| $a_{36}$ | Small classroom use. | Same as above. |
| $a_{37}$ | Office space use. | One square foot. |
| $a_{38}$ | Teaching budget funds. | One dollar per year. |
| $a_{39}$ | "Theory" research funds. | Same as above. |
| $a_{40}$ | "Methods" research funds. | Same as above. |

Table 5. (continued).

| Commodity number | Product description | Unit of measurement |
|---|---|---|
| $a_{41}$ | "Applications" research funds. | One dollar per year. |
| $a_{42}$ | Master's candidates. | One new candidate per year. |
| $a_{43}$ | Ph.D. candidates. | Same as above. |
| $a_{44}$ | "Theory" faculty members. | One faculty member. |
| $a_{45}$ | "Methods" faculty members. | Same as above. |
| $a_{46}$ | "Applications" faculty members. | Same as above. |

Table 6. Research activities.

| Activity numbers | Theory research activities | | | | Methods research activities | | | | | Applications research activities | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $a_4$ | $a_5$ | $a_6$ | $a_7$ | $a_8$ | $a_9$ | $a_{10}$ | $a_{11}$ | $a_{12}$ | $a_{13}$ | $a_{14}$ | $a_{15}$ | $a_{16}$ | a[illegible] |
| Commodity numbers | | | | | | | | | | | | | | |
| $a_8$ | -2.0 | -1.75 | -1.75 | -1.5 | | | | | | | | | | |
| $a_9$ | | | | | -3.0 | -3.0 | -2.25 | -1.75 | -1.5 | | | | | |
| $a_{10}$ | | | | | | | | | | -3.0 | -2.25 | -3.0 | -2.0 | -1.5 |
| $a_{16}$ | 1.0 | | 2.0 | | | | | | | | | | | |
| $a_{17}$ | | | | | 3.0 | | 4.0 | | | | | | | |
| $a_{18}$ | | | | | | | | | | 2.0 | 3.0 | | | |
| $a_{19}$ | 2.0 | 1.5 | | | | | | | | | | | | |
| $a_{20}$ | | | | | 4.0 | 7.0 | | | | | | | | |
| $a_{21}$ | | | | | | | | | | 3.0 | | 4.0 | | |
| $a_{22}$ | .2 | .2 | .2 | .2 | | | | | | | | | | |
| $a_{23}$ | | | | | 1.0 | .75 | .75 | .75 | .5 | | | | | |
| $a_{24}$ | | | | | | | | | | .5 | .25 | .5 | .5 | .125 |
| $a_{25}$ | 1 | 1 | 1 | 1 | | | | | | | | | | |
| $a_{26}$ | | | | | 1 | 1 | 1 | 1 | 1 | | | | | |
| $a_{27}$ | | | | | | | | | | 1 | 1 | 1 | 1 | 1 |
| $a_{39}$ | 500 | 700 | 700 | 1200 | | | | | | | | | | |
| $a_{40}$ | | | | | 2400 | 2800 | 3000 | 5000 | 2000 | | | | | |
| $a_{41}$ | | | | | | | | | | 2400 | 2250 | 4000 | 4000 | 500 |

Table 7. "Degree" activities.

| Activity number | $a_{18}$ | $a_{19}$ | $a_{20}$ |
|---|---|---|---|
| Commodity number | | | |
| $a_1$ | -1 | | |
| $a_2$ | 6 | | |
| $a_3$ | 5 | 1 | |
| $a_4$ | | 6 | 7 |
| $a_5$ | | 4 | 6 |
| $a_{28}$ | 13 | | |
| $a_{29}$ | | 6 | 10 |
| $a_{30}$ | | 4 | 8 |
| $a_{31}$ | | 1 | 3 |
| $a_{33}$ | | 2 | 4 |
| $a_{42}$ | | 1 | |
| $a_{43}$ | | | 1 |

Table 8. Advising, teaching, and thesis supervision.

| | Advising activities | | Undergraduate teaching | | | Teaching activities (applications) | | | Teaching activities (methods) | | Teaching (theory) | | Thesis supervision |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $a_{21}$ | $a_{22}$ | $a_{23}$ | $a_{24}$ | $a_{25}$ | $a_{26}$ | $a_{27}$ | $a_{28}$ | $a_{29}$ | $a_{30}$ | $a_{31}$ | $a_{32}$ | $a_{33}$ |
| $a_2$ | | | -40.0 | -40.0 | -40.0 | | | | | | | | |
| $a_3$ | | | | | | -35.0 | -35.0 | -35.0 | | | | | |
| $a_4$ | | | | | | | | | -25.0 | -25.0 | | | |
| $a_5$ | | | | | | | | | | | -20.0 | -20.0 | |
| $a_6$ | 1.0 | | | | | .85 | .95 | 1.05 | .15 | | | | .2 |
| $a_7$ | | 1.0 | | | | | | | .90 | 1.05 | .20 | | .4 |
| $a_{11}$ | | | | | | | | | | | .85 | 1.05 | .3 |
| $a_{12}$ | | | .15 | .15 | .20 | | | | | | | | |
| $a_{13}$ | | | 2.5 | 3.0 | 2.5 | | | | | | | | |
| $a_{14}$ | | | .5 | .45 | .45 | .2 | .1 | | | | | | |
| $a_{15}$ | 0.1 | 0.1 | .02 | .02 | .02 | .02 | .02 | .02 | .02 | .02 | .02 | .02 | .02 |
| $a_{28}$ | -60.0 | | | | | | | | | | | | |
| $a_{29}$ | | -100.0 | | | | | | | | | | | |
| $a_{30}$ | | | | | | | | | | | | | -6.5 |
| $a_{32}$ | | | | | | | | | | | | | .05 |
| $a_{34}$ | | | | | | | | | | | | | .10 |
| $a_{35}$ | | | .5 | | .5 | | | | | | | | |
| $a_{36}$ | | | 3.0 | 5.0 | 3.0 | 3.0 | 3.0 | 3.0 | 3.0 | 3.0 | 3.0 | 3.0 | |

Table 9. Graduate student appointments.

| Activity numbers | Masters' candidate appointments $a_{34}$ | $a_{35}$ | $a_{36}$ | $a_{37}$ | Ph.D. candidate appointments $a_{38}$ | $a_{39}$ | $a_{40}$ | $a_{41}$ | $a_{42}$ |
|---|---|---|---|---|---|---|---|---|---|
| Commodity numbers | | | | | | | | | |
| $a_{13}$ | | | -18.0 | | | | | | -3.0 |
| $a_{14}$ | | | | | | | | -6.0 | 1 |
| $a_{16}$ | -1.0 | | | | | | | | |
| $a_{17}$ | | -1.0 | | | | | | | |
| $a_{18}$ | | | -1.0 | | | | | | |
| $a_{19}$ | | | | | -1.0 | | | | |
| $a_{20}$ | | | | | | -1.0 | | | |
| $a_{21}$ | | | | | | | -1.0 | | |
| $a_{37}$ | 40 | 40 | 40 | 40 | 47 | 47 | 47 | 60 | |
| $a_{38}$ | | | | 2800 | | | | 3350 | |
| $a_{39}$ | 2700 | | | | 3000 | | | | |
| $a_{40}$ | | 2700 | | | | 3000 | | | |
| $a_{41}$ | | | 2700 | | | | 3000 | | |
| $a_{42}$ | -.5 | -.5 | -.5 | -.5 | | | | | |
| $a_{43}$ | | | | | -1/3 | -1/3 | -1/3 | -1/4 | |

Table 10. Secretarial activities.

| Activity numbers | $a_{43}$ | $a_{44}$ | $a_{45}$ | $a_{46}$ |
|---|---|---|---|---|
| Commodity numbers | | | | |
| $a_{15}$ | | | | -1 |
| $a_{22}$ | -1 | | | |
| $a_{23}$ | | -1 | | |
| $a_{24}$ | | | -1 | |
| $a_{37}$ | 140 | 140 | 140 | 140 |
| $a_{38}$ | | | | 3400 |
| $a_{39}$ | 3400 | | | |
| $a_{40}$ | | 3400 | | |
| $a_{41}$ | | | 3400 | |

Table 11. Faculty allocation activities.

| Activity numbers | Theory faculty | | | | Methods faculty | | | | Applications faculty | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | $a_{47}$ | $a_{48}$ | $a_{49}$ | $a_{50}$ | $a_{51}$ | $a_{52}$ | $a_{53}$ | $a_{54}$ | $a_{55}$ | $a_{56}$ | $a_{57}$ |
| Commodity numbers | | | | | | | | | | | |
| $a_6$ | | | | | | | -3.0 | | -7.0 | -4.0 | |
| $a_7$ | | | -3.0 | | -7.0 | -4.0 | -4.0 | | | | |
| $a_{11}$ | -7.0 | -4.0 | -4.0 | | | | | | | | |
| $a_{12}$ | | -3.0 | | | | -3.0 | | | | -3.0 | |
| $a_{25}$ | -0.34 | -0.34 | -0.34 | -1.0 | | | | | | | |
| $a_{26}$ | | | | | -0.34 | -0.34 | -0.34 | -1.0 | | | |
| $a_{27}$ | | | | | | | | | -0.34 | -0.34 | -1.0 |
| $a_{37}$ | 125 | 125 | 125 | 125 | 125 | 125 | 125 | 125 | 125 | 125 | 125 |
| $a_{38}$ | 8580 | 8580 | 8580 | | 9900 | 9900 | 9900 | | 9240 | 9240 | |
| $a_{39}$ | 4420 | 4420 | 4420 | 13000 | | | | | | | |
| $a_{40}$ | | | | | 5100 | 5100 | 5100 | 15000 | | | |
| $a_{41}$ | | | | | | | | | 4760 | 4760 | 14000 |
| $a_{44}$ | 1.0 | 1.0 | 1.0 | 1.0 | | | | | | | |
| $a_{45}$ | | | | | 1.0 | 1.0 | 1.0 | 1.0 | | | |
| $a_{46}$ | | | | | | | | | 1.0 | 1.0 | 1.0 |

Table 12. Constraints, department A model.

| Commodity constraint[a] | Reason for constraint |
|---|---|
| $y_{a1} \leqq -30.0$ | Fixed target |
| $y_{a2} \leqq -1400.0$ | " " |
| $y_{a3} \leqq -860.85$ | " " |
| $y_{a4} \leqq -85.0$ | " " |
| $y_{a5} \leqq -100.0$ | " " |
| $y_{a6} \leqq -2.89$ | " " |
| $y_{a7} \leqq -2.0$ | " " |
| $A_{ai}x_a - y_{ai} \leqq 0,\ i = 8,9,10$ | Accounting constraints |
| $y_{ai} \leqq 0,\ i = 11, \ldots 30.$ | Intermediate product (accounting) constraints |
| $A_{qi}x_a - y_{ai} \leqq 0\ i = 31,32$ | Accounting constraints |
| $y_{a33} \leqq 40.63$ | Resource allocation granted to department A by the college dean |
| $y_{a34} \leqq 1.25$ | Same as above |
| $y_{a35} \leqq 0.6$ | " " " |
| $y_{a36} \leqq 327.56$ | " " " |
| $y_{a37} \leqq 3226.20$ | " " " |
| $y_{a38} \leqq 133{,}750.62$ | " " " |
| $y_{a39} \leqq 21{,}000$ | Resource constraint imposed by non-university sources of research funds |
| $y_{a40} \leqq 70{,}000$ | Same as above |
| $y_{a41} \leqq 40{,}000$ | " " " |

[a]$y_{ai} = A_{ai}x_a$ for $i = 1, 2 \ldots 46$ where $A_{ai}$ is the ith row of $A_a$.

Table 12. (continued).

| Commodity constraint | Reason for constraint |
| --- | --- |
| $0 \leqq y_{a42} \leqq 2.0$ | Constraints due to students' decisions about graduate school attendance |
| $0 \leqq y_{a43} \leqq 4.0$ | Same as above |
| $y_{a44} = 3.0$ | Numbers of faculty positions authorized by the university president |
| $y_{a45} = 4.5$ | Same as above |
| $y_{a46} = 5.0$ | Same as above |

Table 13. Objective function weights associated with flexible targets, department A.

| Symbols used for flexible target levels[a] | Objective function weights |
|---|---|
| $y_{ao}(= - x_{a20})$[b] | - 4.9 |
| $y_{a00}(= - x_{a19})$[b] | - 2.5 |
| $y_{a8}(= - x_{a1})$ | - 2.0 |
| $y_{a9}(= - x_{a2})$ | - 5.0 |
| $y_{a10}(= - x_{a3})$ | - 4.0 |
| $y_{a31}(= x_{a58})$ | - 0.12 |
| $y_{a32}(= x_{a59})$ | - 1.5 |

[a]The symbols shown in the parentheses were used for computing purposes. They allowed the use of non-negative activity levels and positive objective function weights. The x symbols also are easier to deal with in the discussions to be found in Chapter VII.

[b]For commodities $a_o$ and $a_{oo}$ no constraints were included in the model (see Table 12), but had constraints been included they would have had the form $y_{ao} = - x_{a20}$ and $y_{a00} = - x_{a19}$.

Table 14. Solution to department A model after imposition of quotas

| $x_a$ subscripts | Solution value | $x_a$ subscripts | Solution value |
|---|---|---|---|
| 1 | 1.89 | 31 | 0.0 |
| 2 | 5.46 | 32 | 8.32 |
| 3 | 3.94 | 33 | 12.50 |
| 4 | 0.43 | 34 | 0.43 |
| 5 | 0.59 | 35 | 0.0 |
| 6 | 0.0 | 36 | 0.0 |
| 7 | 0.0 | 37 | 6.55 |
| 8 | 0.0 | 38 | 1.75 |
| 9 | 0.48 | 39 | 3.36 |
| 10 | 0.0 | 40 | 2.16 |
| 11 | 0.0 | 41 | 3.94 |
| 12 | 2.68 | 42 | 0.0 |
| 13 | 0.0 | 43 | 0.20 |
| 14 | 0.0 | 44 | 1.70 |
| 15 | 0.54 | 45 | 0.85 |
| 16 | 1.16 | 46 | 3.0 |
| 17 | 0.0 | 47 | 0.16 |
| 18 | 30.0 | 48 | 1.98 |
| 19 | 5.49 | 49 | 0.86 |
| 20 | 7.41 | 50 | 0.0 |
| 21 | 6.5 | 51 | 1.50 |
| 22 | 1.07 | 52 | 0.0 |
| 23 | 1.20 | 53 | 0.53 |
| 24 | 38.30 | 54 | 2.47 |
| 25 | 0.0 | 55 | 5.0 |
| 26 | 29.04 | 56 | 0.0 |
| 27 | 0.0 | 57 | 0.0 |
| 28 | 0.0 | 58 | 0.63 |
| 29 | 0.0 | 59 | 27.72 |
| 30 | 6.79 | | |

Table 15. Solution to the dual of the department A model.

| Variable | Solution value[a] | Variable | Solution value[a] |
|---|---|---|---|
| $v_{a1}$ | .47 (.855) | $v_{a24}$ | 1.249 (1.737) |
| $v_{a2}$ | .042 (.032) | $v_{a25}$ | 2.814 (2.862) |
| $v_{a3}$ | .033 (.047) | $v_{a26}$ | 6.193 (6.024) |
| $v_{a4}$ | .047 (.074) | $v_{a27}$ | 5.980 (5.855) |
| $v_{a5}$ | .059 (.092) | $v_{a28}$ | .004 (.033) |
| $v_{a6}$ | .204 (1.571) | $v_{a29}$ | .003 (.032) |
| $v_{a7}$ | .204 (1.571) | $v_{a30}$ | .453 (.266) |
| $v_{a8}$ | 2.0 (2.0) | $v_{a31}$ | .120 (.120) |
| $v_{a9}$ | 5.0 (5.0) | $v_{a32}$ | 1.500 (1.500) |
| $v_{a10}$ | 4.0 (4.0) | $v_{a33}$ | 0.0 (.123) |
| $v_{a11}$ | .204 (1.571) | $v_{a34}$ | 26.804 (1.582) |
| $v_{a12}$ | .204 (1.571) | $v_{a35}$ | 1.277 (.252) |
| $v_{a13}$ | .013 (.169) | $v_{a36}$ | 0.317 (.036) |
| $v_{a14}$ | .038 (.594) | $100v_{a37}$ | 0.045 (.466) |
| $v_{a15}$ | .321 (4.12) | $1000v_{a38}$ | 0.076 (1.020) |
| $v_{a16}$ | .359 (.381) | $1000v_{a39}$ | 0.126 (.076) |
| $v_{a17}$ | .931 (.874) | $1000v_{a40}$ | 0.345 (.311) |
| $v_{a18}$ | .779 (.755) | $1000v_{a41}$ | 0.349 (.319) |
| $v_{a19}$ | .332 (.269) | $v_{a42}$ | 0.0 (.020) |
| $v_{a20}$ | .988 (.975) | $v_{a43}$ | 0.203 (529) |
| $v_{a21}$ | 1.000 (1.000) | $v_{a44}$ | 1.116 (2.301) |
| $v_{a22}$ | .493 (.909) | $v_{a45}$ | 0.965 (.779) |
| $v_{a23}$ | 1.236 (1.709) | $v_{a46}$ | 1.041 (1.461) |

[a]The values in parentheses are elements of the vector which solves the dual of 7.2 (version 2 of the college model.

# XIII. APPENDIX B

Table 16. Final products, department B.

| Commodity number | Product description | Units of measurement |
|---|---|---|
| $b_o$[a] | Ph.D. degrees awarded by department B. | One degree per academic year. |
| $b_{oo}$[a] | Masters' degrees awarded by department B. | Same as above. |
| $b_1$ | Training of undergraduate students majoring in discipline B. | One undergraduate unit per year. One unit is the amount of instruction (and departmental administration) in discipline B which is required to produce one Bachelors' degree (in discipline B) per year. |
| $b_2$ | Instructions in the principles (I) of discipline B. | One instructional unit per year. An instructional unit is one (3 quarter hours) course taught to one student. |
| $b_3$ | Instruction in the principles (II) of discipline B. | Same as above. |
| $b_4$ | Advanced undergraduate (or minor graduate) instruction in the general theory of discipline B. | Same as above. |
| $b_5$ | Advanced undergraduate (or minor graduate) instruction in the applied theories of discipline B. | Same as above |

[a]Commodities $b_o$ and $b_{oo}$ are commodities for which no constraints have been included in the model.

Table 16. (Continued).

| Commodity number | Product description | Units of measurement |
|---|---|---|
| $b_6$ | Graduate instruction in the general theory of discipline B. | One instructional unit per year. An instructional unit is one (3 quarter hours) course taught to one student. |
| $b_7$ | Graduate instruction in the applied theories of discipline B. | Same as above. |
| $b_8$ | Faculty teaching services (advanced undergraduate or minor graduate teaching) rendered by "general theory" faculty members. | One unit of teaching per year. A unit of teaching is the amount required to teach (including preparation) one (3 quarter hours) course. |
| $b_9$ | Faculty teaching services (advanced undergraduate or minor graduate teaching) rendered by "applied theories" faculty members. | Same as above. |
| $b_{10}$ | Faculty teaching services (graduate teaching) rendered by "general theory" faculty members. | Same as above. |
| $b_{11}$ | Faculty teaching services (graduate teaching) rendered by "applied theories" faculty members. | Same as above. |
| $b_{12}$ | Publications (non-project research) by "general theory" faculty members. | One publication (10 published pages) per year. |

Table 16. (Continued).

| Commodity number | Product description | Units of measurement |
|---|---|---|
| $b_{13}$ | Publications (project research) by "general theory" faculty members. | One publication (10 published pages) per year. |
| $b_{14}$ | Publications (project research) by "applied theories" faculty members. | Same as above. |
| $b_{15}$ | Publications (non-project research) by "applied theories" faculty members. | Same as above. |

Table 17. Intermediate products, department B.

| Commodity numbers | Product description | Units of measurement |
|---|---|---|
| $b_{16}$ | Faculty teaching services (principles I and II). | Same as for $b_8$. |
| $b_{17}$ | Teaching services provided by graduate teaching assistants. | Same as for $b_8$. |
| $b_{18}$ | Teaching services provided by graduate instructors. | Same as for $b_8$. |
| $b_{19}$ | Secretarial and clerical assistance used by teaching activities. | One woman year (9-10 months) per year. |
| $b_{20}$ | Secretarial and clerical assistance used by "general theory" research activities. | Same as above. |
| $b_{21}$ | Secretarial and clerical assistance used by "applied theories" research activities. | Same as above. |
| $b_{22}$ | Research services provided by "general theory" faculty members for non-project research. | One man year (9-10 months) per year. |
| $b_{23}$ | Research services provided by "general theories" faculty members for project research. | Same as above. |
| $b_{24}$ | Research services provided by "applied theories" faculty members for project research. | Same as above. |

Table 17. (Continued).

| Commodity numbers | Product description | Units of measurement |
|---|---|---|
| $b_{25}$ | Research services provided by "applied theories" faculty members for non-project research. | One man year (9-10 months) per year. |
| $b_{26}$ | Counselling and advising of undergraduate students. | One unit of counselling per year. One unit is the amount of advising required per (discipline B major) student per quarter. |
| $b_{27}$ | Counselling and advising of graduate students. | Same as above. |
| $b_{28}$ | Thesis (and related) supervision of graduate students. | The supervision of 3 thesis credit hours per year. |

Table 18. Primary products, department B.

| Commodity numbers | Product description | Units of measurement |
|---|---|---|
| $b_{29}$ | Instruction provided by department B's students by departments outside of B's college. | Same as for $b_2$. |
| $b_{30}$ | Teaching services provided to department B by departments outisde of B's college. | Same as for $b_8$. |
| $b_{31}$ | Small classroom use. | One classroom unit per year. One unit is the amount (number of hours) of classroom use required to conduct a class meeting one hour per week for one quarter. |
| $b_{32}$ | Office space use. | One square foot. |
| $b_{33}$ | Teaching budget funds. | One dollar per year. |
| $b_{34}$ | Project research funds (general theory research). | Same as above. |
| $b_{35}$ | Project research funds (applied theories research). | Same as above. |
| $b_{36}$ | Masters' candidate. | One new candidate per year. |
| $b_{37}$ | Ph.D. candidate. | Same as above. |
| $b_{38}$ | "General theory" faculty members. | One faculty member. |
| $b_{39}$ | "Applied theory" faculty members. | Same as above. |

Table 19. Research activities.

| Activity numbers | Special theory research activities | | | | General theory research activities | | | |
|---|---|---|---|---|---|---|---|---|
| | $b_5$ | $b_6$ | $b_7$ | $b_8$ | $b_9$ | $b_{10}$ | $b_{11}$ | $b_{12}$ |
| Commodity numbers | | | | | | | | |
| $b_{12}$ | -.8 | -1.0 | | | | | | |
| $b_{13}$ | | | -1.2 | -1.25 | | | | |
| $b_{14}$ | | | | | -1.5 | -1.6 | | |
| $b_{15}$ | | | | | | | -1.0 | -1.1 |
| $b_{19}$ | .1 | .2 | | | | | .1 | .2 |
| $b_{20}$ | | | .1 | .2 | | | | |
| $b_{21}$ | | | | | .2 | .3 | | |
| $b_{22}$ | 1.0 | 1.0 | | | | | | |
| $b_{23}$ | | | 1.0 | 1.0 | | | | |
| $b_{24}$ | | | | | 1.0 | 1.0 | | |
| $b_{25}$ | | | | | | | 1.0 | 1.0 |
| $b_{34}$ | | | 500 | 750 | | | | |
| $b_{35}$ | | | | | 400 | 1000 | | |

Table 20. "Degree" activities.

| Activity numbers | $b_{13}$ | $b_{14}$ | $b_{15}$ |
|---|---|---|---|
| Commodity numbers | | | |
| $b_1$ | -1 | | |
| $b_2$ | 1 | | |
| $b_3$ | 4 | | |
| $b_4$ | 6 | | |
| $b_5$ | 5 | | |
| $b_6$ | | 6 | 7 |
| $b_7$ | | 4 | 6 |
| $b_{26}$ | 13 | | |
| $b_{27}$ | | 6 | 10 |
| $b_{28}$ | | 4 | 8 |
| $b_{29}$ | | 3 | 7 |
| $b_{36}$ | | 1 | |
| $b_{37}$ | | | 1 |

Table 21. Advising, teaching, and thesis supervision activities.

| Activity numbers | Undergraduate advising $b_{16}$ | $b_{17}$ | Undergraduate teaching (principles) activities $b_{18}$ | $b_{19}$ | $b_{20}$ | $b_{21}$ | $b_{22}$ | $b_{23}$ | $b_{24}$ |
|---|---|---|---|---|---|---|---|---|---|
| Commodity numbers | | | | | | | | | |
| $b_2$ | | | -35.0 | -45.0 | -35.0 | -35.0 | | | |
| $b_3$ | | | | | | | -35.0 | -45.0 | -35.0 |
| $b_4$ | | | | | | | | | |
| $b_5$ | | | | | | | | | |
| $b_6$ | | | | | | | | | |
| $b_7$ | | | | | | | | | |
| $b_8$ | 1 | | | | | | | | |
| $b_9$ | | 1 | | | | | | | |
| $b_{10}$ | | | | | | | | | |
| $b_{11}$ | | | | | | | | | |
| $b_{16}$ | | | .25 | 1.05 | .25 | .35 | .45 | 1.05 | .45 |
| $b_{17}$ | | | .35 | | | .75 | .15 | | |
| $b_{18}$ | | | .50 | | .85 | | .45 | | .60 |
| $b_{19}$ | .1 | .1 | .02 | .02 | .02 | .02 | .02 | .02 | .02 |
| $b_{26}$ | -75.0 | -75.0 | | | | | | | |
| $b_{27}$ | | | | | | | | | |
| $b_{28}$ | | | | | | | | | |
| $b_{30}$ | | | | | | | | | |
| $b_{31}$ | | | 3 | 3 | 3 | 3 | 3 | 3 | 3 |

Table 21. (Continued).

| Activity numbers | Undergraduate teaching $b_{25}$ | $b_{26}$ | Graduate advising $b_{27}$ | $b_{28}$ | Graduate teaching $b_{29}$ | $b_{30}$ | Thesis supervision $b_{31}$ |
|---|---|---|---|---|---|---|---|
| Commodity numbers | | | | | | | |
| $b_2$ | | | | | | | |
| $b_3$ | | | | | | | |
| $b_4$ | -30.0 | | | | | | |
| $b_5$ | | -30.0 | | | | | |
| $b_6$ | | | | | -15.0 | | |
| $b_7$ | | | | | | -15.0 | |
| $b_8$ | 1.05 | | | | | | |
| $b_9$ | | 1.05 | | | | | |
| $b_{10}$ | | | 1 | | 1.05 | | .45 |
| $b_{11}$ | | | | 1 | | 1.05 | .5 |
| $b_{16}$ | | | | | | | |
| $b_{17}$ | | | | | | | |
| $b_{18}$ | | | | | | | |
| $b_{19}$ | .02 | .02 | .4 | .4 | .02 | .02 | .02 |
| $b_{26}$ | | | | | | | |
| $b_{27}$ | | | -100.0 | -100.0 | | | |
| $b_{28}$ | | | | | | | -7.0 |
| $b_{30}$ | | | | | | | .10 |
| $b_{31}$ | 3 | 3 | | | 3 | 3 | |

Table 22. Graduate student appointments activities.

| Activity numbers | $b_{32}$ | $b_{33}$ | $b_{34}$ | $b_{35}$ |
|---|---|---|---|---|
| Commodity numbers | | | | |
| $b_{17}$ | -5 | -5 | | |
| $b_{18}$ | | | -5 | -5 |
| $b_{32}$ | 40 | 40 | 60 | 60 |
| $b_{33}$ | 2750 | 2750 | 3200 | 3200 |
| $b_{36}$ | -.6 | -.5 | | |
| $b_{37}$ | | | -.3 | -.25 |

Table 23. Secretarial activities.

| Activity numbers | $b_{36}$ | $b_{37}$ | $b_{38}$ |
|---|---|---|---|
| Commodity numbers | | | |
| $b_{19}$ | -1 | | |
| $b_{20}$ | | -1 | |
| $b_{21}$ | | | -1 |
| $b_{32}$ | 140 | 140 | 140 |
| $b_{33}$ | 3400 | | |
| $b_{35}$ | | 3400 | |
| $b_{36}$ | | | 3400 |

Table 24. Faculty allocation activities.

| Activity numbers | "General theory" faculty allocation activities | | | | | "Applied theories" faculty allocation activities | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | $b_{39}$ | $b_{40}$ | $b_{41}$ | $b_{42}$ | $b_{43}$ | $b_{44}$ | $b_{45}$ | $b_{46}$ | $b_{47}$ | $b_{48}$ |
| Commodity numbers | | | | | | | | | | |
| $b_8$ | | -5.0 | -3.0 | -1.0 | | | | | | |
| $b_9$ | | | | | | | -5.0 | -3.0 | -1.0 | |
| $b_{10}$ | | -2.0 | -1.0 | -4.0 | | | | | | |
| $b_{11}$ | | | | | | | -2.0 | -1.0 | -4.0 | |
| $b_{16}$ | | -2.0 | -5.0 | -4.0 | | | -2.0 | -5.0 | -4.0 | |
| $b_{22}$ | 1 | -.2 | -.2 | -.2 | -1.0 | | | | | |
| $b_{23}$ | -1 | | | | | | | | | |
| $b_{24}$ | | | | | | -1 | | | | |
| $b_{25}$ | | | | | | 1 | -.2 | -.2 | -.2 | -1.0 |
| $b_{32}$ | | 125 | 125 | 125 | 125 | | 125 | 125 | 125 | 125 |
| $b_{33}$ | -13000 | 13000 | 13000 | 13000 | 13000 | -13500 | 13500 | 13500 | 13500 | 13500 |
| $b_{34}$ | 13000 | | | | | | | | | |
| $b_{35}$ | | | | | | 13500 | | | | |
| $b_{38}$ | | 1 | 1 | 1 | 1 | | | | | |
| $b_{39}$ | | | | | | | 1 | 1 | 1 | 1 |

Table 25. Constraints, department B.

| Commodity constraints[a] | Reason for constraint |
|---|---|
| $y_{b1} \leqq -120.0$ | Fixed target |
| $y_{b2} \leqq -6900.$ | " " |
| $y_{b3} \leqq -4700$ | " " |
| $y_{b4} \leqq -453.83$ | " " |
| $y_{b5} \leqq -900$ | " " |
| $y_{b6} \leqq -200.62$ | " " |
| $y_{b7} \leqq -225$ | " " |
| $y_{b8} \leqq -1.98$ | " " |
| $y_{b9} \leqq -1.0$ | " " |
| $y_{b10} \leqq -4.0$ | " " |
| $y_{b11} \leqq -3.0$ | " " |
| $A_{bi}x_b - y_{bi} \leqq 0 \quad i = 12,13,14,15$ | Accounting constraint |
| $y_{bi} \leqq 0 \quad i = 16,17,\ldots.28$ | Intermediate product (accounting) constraint |
| $A_{bi}x_b - y_{bi} \leqq 0 \quad i = 29,30$ | |
| $y_{b31} \leqq 1439.73$ | Resource allocation granted to department A by the college dean |
| $y_{b32} \leqq 8805.89$ | Same as above |
| $y_{b33} \leqq 603{,}863.63$ | " " " |
| $y_{b34} \leqq 20{,}000$ | Resource constraint imposed by non-university sources of research funds. |

[a] $y_{bi} = A_{bi}x_b$ for $i = 1,2,3\ldots39$ where $A_{bi}$ is the ith row of $A_b$.

Table 25. (Continued).

| Commodity constraints[a] | Reason for constraints |
|---|---|
| $y_{b35} \leqq 25{,}000$ | Resource constraints imposed by non-university sources of research funds. |
| $0 \leqq y_{b36} \leqq 4.0$ | Constraints due to students' decisions about graduate school attendance |
| $0 \leqq y_{b37} \leqq 8.0$ | Same as above |
| $y_{b38} = 16.0$ | Numbers of faculty positions authorized by the university president. |
| $y_{b39} = 17.5$ | Same as above |

Table 26. Objective function weights associated with flexible targets, department B.

| Symbols used for flexible target levels[a] | Objective function weights |
|---|---|
| $y_{bo}(= - x_{b15})$[b] | -4.75 |
| $y_{boo}(= - x_{b14})$[b] | -2.45 |
| $y_{b12}(= - x_{b1})$ | -1.5 |
| $y_{b13}(= - x_{b2})$ | -2.25 |
| $y_{b14}(= - x_{b3})$ | -2.5 |
| $y_{b15}(= - x_{b4})$ | -2.0 |
| $y_{b29}(= x_{49})$ | -0.12 |
| $y_{b30}(= x_{50})$ | -1.5 |

[a]The symbols shown in the parentheses were used for computing purposes since they allowed the use of only non-negative activity levels and the use of positive objective function weights. x symbols are also easier to deal with in the discussions to be found in Chapter VII.

[b]For commodities $b_o$ and $b_{oo}$ no constraints were included in the model (see Table 25), but had constraints been included they would have had the form $y_{bo} = - x_{b15}$ and $y_{boo} = - x_{b14}$.

Table 27. Solution to department B model after quotas are imposed.

| $x_b$ subscript | Solution value | $x_b$ subscript | Solution value |
|---|---|---|---|
| 1 | 1.40 | 26 | 50.00 |
| 2 | 1.73 | 27 | 1.44 |
| 3 | 2.57 | 28 | 0.0 |
| 4 | 1.79 | 29 | 21.20 |
| 5 | 1.75 | 30 | 21.01 |
| 6 | 0.0 | 31 | 15.40 |
| 7 | 1.45 | 32 | 0.0 |
| 8 | 0.0 | 33 | 18.48 |
| 9 | 1.71 | 34 | 0.0 |
| 10 | 0.0 | 35 | 33.38 |
| 11 | 1.79 | 36 | 12.92 |
| 12 | 0.0 | 37 | 0.14 |
| 13 | 120.0 | 38 | 0.34 |
| 14 | 9.24 | 39 | 1.45 |
| 15 | 8.86 | 40 | 10.61 |
| 16 | 20.8 | 41 | 2.72 |
| 17 | 0.0 | 42 | 2.67 |
| 18 | 200.57 | 43 | 0.0 |
| 19 | 0.0 | 44 | 1.71 |
| 20 | 0.0 | 45 | 4.19 |
| 21 | 0.0 | 46 | 9.62 |
| 22 | 148.00 | 47 | 3.69 |
| 23 | 0.0 | 48 | 0.0 |
| 24 | 0.0 | 49 | 89.71 |
| 25 | 39.13 | 50 | 2.31 |

Table 28. Solution to the dual of department B model.

| Variable | Solution value[a] | Variable | Solution value[a] |
|---|---|---|---|
| $v_{b1}$ | 1.135 (1.208) | $v_{b20}$ | 3.369 (4.363) |
| $v_{b2}$ | 0.019 (0.033) | $v_{b21}$ | 3.338 (4.337) |
| $v_{b3}$ | 0.027 (0.038) | $v_{b22}$ | .900 (0.788) |
| $v_{b4}$ | 0.060 (0.062) | $v_{b23}$ | 2.309 (1.718) |
| $v_{b5}$ | 0.060 (0.062) | $v_{b24}$ | 3.043 (2.449) |
| $v_{b6}$ | 0.121 (0.123) | $v_{b25}$ | 1.700 (1.588) |
| $v_{b7}$ | 0.121 (0.123) | $v_{b26}$ | 0.026 (0.027) |
| $v_{b8}$ | 1.669 (1.582) | $v_{b27}$ | 0.029 (0.032) |
| $v_{b9}$ | 1.669 (1.582) | $v_{b28}$ | 0.257 (0.248) |
| $v_{b10}$ | 1.669 (1.582) | $v_{b29}$ | .120 (0.120) |
| $v_{b11}$ | 1.669 (1.582) | $v_{b30}$ | 1.5 (1.5) |
| $v_{b12}$ | 1.500 (1.500) | $v_{b31}$ | 0.0 (0.036) |
| $v_{b13}$ | 2.250 (1.500) | $100v_{b32}$ | 2.143 (0.466) |
| $v_{b14}$ | 2.500 (2.500) | $1000v_{b33}$ | 0.0 (1.020) |
| $v_{b15}$ | 2.000 (2.000) | $1000v_{b34}$ | 0.108 (1.092) |
| $v_{b16}$ | 1.669 (1.582) | $1000v_{b35}$ | 0.099 (1.084) |
| $v_{b17}$ | .203 (0.631) | $v_{b36}$ | -0.3166(-0.329) |
| $v_{b18}$ | .257 (0.709) | $v_{b37}$ | 0.0 (0.0) |
| $v_{b19}$ | 3.000 (4.120) | $v_{b38}$ | 12.524 (0.552) |
| | | $v_{b39}$ | 12.684 (0.202) |

[a]The values in parentheses are elements of the vector which solves the dual of 7.2 (version 2 of the college model).

# XIV. APPENDIX C

Table 29. Bachelors' degree activity.

| Activity number | | | $c_{48}$ |
|---|---|---|---|
| Commodity number (dept. models) | Commodity numbers (version one) | Commodity numbers (version two) | |
| — | $c_1$[a] | — | -1 |
| $a_1$ | $c_{39}$ | 11 | .12 |
| $b_1$ | $c_{40}$ | 49 | .48 |
| $e_1$ | $c_{41}$ | 82 | .40 |
| $a_2$ | $c_2$ | 12 | 2 |
| $a_3$ | $c_3$ | 1 | 1.5 |
| $b_2$ | $c_4$ | 50 | 1 |
| $b_3$ | $c_5$ | 51 | .5 |
| $b_4$ | $c_6$ | 2 | .5 |
| $b_5$ | $c_7$ | 52 | .3 |
| $e_2$ | $c_8$ | 83 | 1 |
| $e_3$ | $c_9$ | 84 | .7 |
| $e_4$ | $c_{10}$ | 85 | .2 |
| $e_5$ | $c_{11}$ | 86 | .3 |
| — | $c_{99}$[b] | — | 35 |

[a]Commodity $c_1$ is Bachelors' degrees. The unit of measurement is one degree per year.

[b]Commodity $c_{99}$ is instruction received by undergraduate students outside of the college. The unit of measurement is one (3 quarter hours) instructional unit per year.

Table 30. Cross reference, college commodity numbers (version 1) and departmental commodity numbers, final products (from college viewpoint).

| | Commodity numbers used in the model of | | | |
|---|---|---|---|---|
| | Dept. A | Dept. B | Dept. D | Econ. dept. |
| College commodity numbers | | | | |
| 1[a] | | | | |
| 2 | 2 | | | |
| 3 | 3 | | | 38 |
| 4 | | 2 | | |
| 5 | | 3 | | |
| 6 | | 4 | | 39 |
| 7 | | 5 | | |
| 8 | | | | 2 |
| 9 | | | | 3 |
| 10 | | | | 4 |
| 11 | | | | 5 |
| 12 | 4 | | | |
| 13 | 5 | | | |
| 14 | 33 | 6 | | |
| 15 | | 7 | | |
| 16 | | | | 6 |
| 17 | | | | 7 |
| 18 | | | | 8 |
| 19 | 6 | | | |
| 20 | 7 | | | |
| 21 | | 8 | | |
| 22 | | 9 | | |
| 23 | 34 | 10 | | |
| 24 | | 11 | | |
| 25 | | | | 9 |
| 26 | | | | 10 |
| 27 | | | | 11 |
| 28 | 8 | | | |

[a]College commodity one is B.S. degrees. The unit of measurement is one degree per year.

Table 30. (Continued).

| | Commodity numbers used in the model of | | | |
|---|---|---|---|---|
| | Dept. A | Dept. B | Dept. D | Econ. dept. |
| College commodity numbers | | | | |
| 29 | 9 | | | |
| 30 | 10 | | | |
| 31 | | 12 | | |
| 32 | | 13 | | |
| 33 | | 14 | | |
| 34 | | 15 | | |
| 35 | | | | 12 |
| 36 | | | | 13 |
| 37 | | | | 14 |
| 38 | | | | 15 |

Table 31. Cross reference, college commodity numbers (version 1) and departmental commodity numbers, intermediate products (from college viewpoint).

| College commodity numbers | Commodity numbers used in the model of Dept. A | Dept. B | Dept. "D" | Econ. dept. |
|---|---|---|---|---|
| 39 | 1 | | | |
| 40 | | 1 | | |
| 41 | | | | 1 |
| 42 | 11 | | | |
| 43 | 12 | | | |
| 44 | 13 | | | |
| 45 | 14 | | | |
| 46 | 15 | | | |
| 47 | | 16 | | |
| 48 | | 17 | | |
| 49 | | 18 | | |
| 50 | | 19 | | |
| 51 | | | | 16 |
| 52 | | | | 17 |
| 53 | | | | 18 |
| 54 | | | | 19 |
| 55 | | | | 20 |
| 56 | | | | 21 |
| 57 | | | | 22 |
| 58 | 16 | | | |
| 59 | 17 | | | |
| 60 | 18 | | | |
| 61 | 19 | | | |
| 62 | 20 | | | |
| 63 | 21 | | | |
| 64 | 22 | | | |
| 65 | 23 | | | |
| 66 | 24 | | | |
| 67 | 25 | | | |
| 68 | 26 | | | |
| 69 | 27 | | | |
| 70 | | 20 | | |

Table 31. (Continued).

| College commodity number | Commodity numbers used in the model of Dept. A | Dept. B | Dept. "D" | Econ. Dept. |
|---|---|---|---|---|
| 71 | | 21 | | |
| 72 | | 22 | | |
| 73 | | 23 | | |
| 74 | | 24 | | |
| 75 | | 25 | | |
| 76 | | | | 23 |
| 77 | | | | 24 |
| 78 | | | | 25 |
| 79 | | | | 26 |
| 80 | | | | 27 |
| 81 | | | | 28 |
| 82 | | | | 29 |
| 83 | | | | 30 |
| 84 | | | | 31 |
| 85 | | | | 32 |
| 86 | | | | 33 |
| 87 | 28 | | | |
| 88 | 29 | | | |
| 89 | | 26 | | |
| 90 | | 27 | | |
| 91 | | | | 34 |
| 92 | | | | 35 |
| 93 | 30 | | | |
| 94 | | 28 | | |
| 95 | | | | 36 |
| 96 | | | | 37 |
| 97 | 36 | 31 | 1 | 43 |

Table 32. Cross reference, college commodity number (version 1) and departmental commodity numbers, primary products (from college viewpoint).

| College commodity numbers | Commodity numbers used in the model of Dept. A | Dept. B | Dept. "D" | Econ. dept. |
|---|---|---|---|---|
| 98[a] | | | | |
| 99 | 31 | | | |
| 100 | | 29 | | |
| 101 | 32 | | | |
| 102 | | 30 | | |
| 103 | 35 | | 2 | 42 |
| 104 | | | 3 | |
| 105 | 37 | 32 | | 44 |
| 106 | 38 | 33 | 4 | 45 |
| 107 | 39 | | | |
| 108 | 40 | | | |
| 109 | 41 | | | |
| 110 | | 34 | | |
| 111 | | 35 | | |
| 112 | | | | 46 |
| 113 | | | | 47 |
| 114 | | | | 48 |
| 115 | 42 | | | |
| 116 | 43 | | | |
| 117 | | 36 | | |
| 118 | | 37 | | |
| 119 | | | | 49 |
| 120 | | | | 50 |
| 121 | | | | 51 |
| 122 | | | | 52 |
| 123 | 44 | | | |
| 124 | 45 | | | |
| 125 | 46 | | | |
| 126 | | 38 | | |

[a]College commodity $a_9$ is teaching provided to the undergraduate students of the college being considered by departments in other colleges. The unit of measurement is one instructional unit per year.

Table 32. (Continued).

| College commodity numbers | Commodity numbers used in the model of Dept. A | Dept. B | Dept. "D" | Econ. dept. |
|---|---|---|---|---|
| 127 | | 39 | | |
| 128 | | | | 53 |
| 129 | | | | 54 |
| 130 | | | | 55 |
| 131 | | | | 56 |
| 132 | | | | 57 |

Table 33. Cross reference between college and departmental activity numbers (version one)

| | College activity no. | Dept. A no. | Dept. B no. | Dept. "D" no. | Dept. E no. | i = |
|---|---|---|---|---|---|---|
| Research | i | i | | | | 1 to 17 |
| activities | i | | i-17 | | | 18 to 29 |
| | i | | | | i-29 | 30 to 47 |
| "Degree" | 48 | | | | | 48 |
| activities | i | i-31 | | | | 49 to 51 |
| | i | | i-39 | | | 52 to 54 |
| | i | | | | i-36 | 55 to 59 |
| Teaching | i | i-39 | | | | 60 to 72 |
| activities | i | | i-57 | | | 73 to 88 |
| | i | | | | i-65 | 89 to 107 |
| Graduate | i | i-57 | | | | 108 to 116 |
| student | i | | i-85 | | | 117 to 120 |
| appointment | i | | | | i-78 | 121 to 135 |
| activities | | | | | | |
| Secretarial | i | i-93 | | | | 136 to 139 |
| activities | i | | i-104 | | | 140 to 142 |
| | i | | | | i-85 | 143 to 146 |
| Faculty | i | i-100 | | | | 147 to 157 |
| activities | i | | i-119 | | | 158 to 167 |
| | i | | | | i-106 | 168 to 188 |
| Space activities | i | | | i-188 | | 189 to 192 |
| Resource importing | i | i-135 | | | | 193 to 194 |
| activities | i | | i-146 | | | 195 to 196 |

Table 34. Constraints, college model (version one).

| Commodity constraint[a] | Reason for constraint |
|---|---|
| $y_{c1} \leqq -250$ | Fixed target |
| $y_{c2} \leqq -900$ | " " |
| $y_{c3} \leqq -350$ | " " |
| $y_{c4} \leqq -6650$ | " " |
| $y_{c5} \leqq -4575$ | " " |
| $y_{c6} \leqq -275$ | " " |
| $y_{c7} \leqq -825$ | " " |
| $y_{c8} \leqq -2750$ | " " |
| $y_{c9} \leqq -525$ | " " |
| $y_{c10} \leqq -150$ | " " |
| $y_{c11} \leqq -225$ | " " |
| $y_{c12} \leqq -85$ | " " |
| $y_{c13} \leqq -100$ | " " |
| $y_{c14} \leqq -160$ | " " |
| $y_{c15} \leqq -225$ | " " |
| $y_{c16} \leqq -15$ | " " |
| $y_{c17} \leqq -40$ | " " |
| $y_{c18} \leqq -75$ | " " |
| $y_{c19} \leqq -.75$ | " " |
| $y_{c20} \leqq -2.$ | " " |
| $y_{c21} \leqq -1.25$ | " " |
| $y_{c22} \leqq -1$ | " " |

[a] $y_{ci} = A_{ci}x_c$ for $i = 1, 2 \ldots 132$.

Table 34. (Continued).

| Commodity constraint[a] | Reason for constraint |
|---|---|
| $y_{c23} \leqq -2.75$ | Fixed target |
| $y_{c24} \leqq -3$ | " " |
| $y_{c25} \leqq -4$ | " " |
| $y_{c26} \leqq -2$ | " " |
| $y_{c27} \leqq -8$ | " " |
| $A_{ci}x_c - y_{ci} \leqq 0,\ i = 28$ to $38$ | Accounting constraint |
| $y_{ci} \leqq 0,\ i = 39$ to $97$ | Intermediate product (accounting) constraint |
| $y_{c98} \leqq 8750$ | Resource allocation granted by university president |
| $A_{ci}x_c - y_{ci} \leqq 0,\ i = 99$ to $102$ | Primary product (accounting) constraint |
| $y_{c103} \leqq 27$ | Resource allocation granted by university president |
| $y_{c104} \leqq 25$ | Same as above |
| $y_{c105} \leqq 19125$ | " " " |
| $y_{c106} \leqq 1{,}095{,}000$ | " " " |
| $y_{c107} \leqq 21{,}000$ | Constraint imposed by non-university sources of research funds |
| $y_{c108} \leqq 70{,}000$ | Resource constraints imposed by non-university sources of research funds. |
| $y_{c109} \leqq 40{,}000$ | Same as above |
| $y_{c110} \leqq 20{,}000$ | " " " |
| $y_{c111} \leqq 25{,}000$ | " " " |
| $y_{c112} \leqq 7200$ | " " " |

Table 34. (Continued).

| Commodity constraints[a] | Reason for constraint |
|---|---|
| $y_{c113} \leqq 80{,}000$ | Resource constraints imposed by non-university sources of research funds. |
| $y_{c114} \leqq 125{,}000$ | Same as above |
| $0 \leqq y_{c115} \leqq 2$ | Constraints due to students' decisions about graduate school attendance |
| $0 \leqq y_{c116} \leqq 4$ | Same as above |
| $0 \leqq y_{c117} \leqq 4$ | " " " |
| $0 \leqq y_{c118} \leqq 8$ | " " " |
| $0 \leqq y_{c119} \leqq 6$ | " " " |
| $0 \leqq y_{c120} \leqq 8$ | " " " |
| $0 \leqq y_{c121} \leqq 3$ | " " " |
| $0 \leqq y_{c122} \leqq 4$ | " " " |
| $y_{c123} = 3$ | Numbers of faculty positions authorized by the university president |
| $y_{c124} = 4.5$ | Same as above |
| $y_{c125} = 5$ | " " " |
| $y_{c126} = 16$ | " " " |
| $y_{c127} = 17.5$ | " " " |
| $y_{c128} = 5$ | " " " |
| $y_{c129} = 2$ | " " " |
| $y_{c130} = 6.5$ | " " " |
| $y_{c131} = 8$ | " " " |
| $y_{c132} = 3$ | " " " |

Table 35. Objective function weights associated with flexible targets, college model (version one)

| Symbols used for flexible target levels[a] | Objective function weights |
|---|---|
| $y_{c28}$ $(-\ x_{c1})$ | - 2 |
| $y_{c29}$ $(-\ x_{c2})$ | - 5 |
| $y_{c30}$ $(-\ x_{c3})$ | - 4 |
| $y_{c31}$ $(-\ x_{c18})$ | - 1.5 |
| $y_{c32}$ $(-\ x_{c19})$ | - 2.25 |
| $y_{c33}$ $(-\ x_{c20})$ | - 2.5 |
| $y_{c34}$ $(-\ x_{c21})$ | - 2.0 |
| $y_{c35}$ $(-\ x_{c30})$ | - 2.0 |
| $y_{c36}$ $(-\ x_{c31})$ | - 2.0 |
| $y_{c37}$ $(-\ x_{c32})$ | - 4.0 |
| $y_{c38}$ $(-\ x_{c33})$ | - 4.0 |
| $x_{c50}$ | 2.5 |
| $x_{c51}$ | 4.9 |
| $x_{c53}$ | 2.45 |
| $x_{c54}$ | 4.75 |
| $x_{c56}$ | 2.4 |
| $x_{c57}$ | 2.4 |
| $x_{c58}$ | 4.8 |
| $x_{c59}$ | 4.8 |
| $y_{c99}$ $(-\ x_{c181})$ | + 0.12 |

[a]The symbols shown in the parentheses were used for computing purposes. They allowed the use of non-negative activity levels and positive objective function weights.

Table 35.— (Continued).

| Symbols used for flexible target levels[a] | Objective function weights |
|---|---|
| $y_{c100}$ (- $x_{c182}$) | 1.5 |
| $y_{c101}$ (- $x_{c183}$) | 0.12 |
| $y_{c102}$ (- $x_{c184}$) | 1.5 |

Table 36. Cross reference college commodity numbers (version two) and departmental commodity numbers.

| College commodity numbers | Dept. A commodity numbers | Dept. B commodity numbers | Dept. "D" commodity numbers | Dept. E commodity numbers |
|---|---|---|---|---|
| 1 | 3 | | | 38 |
| 2 | | 4 | | 39 |
| 3 | 33 | 6 | | |
| 4 | 6 | | | 40 |
| 5 | | 8 | | 41 |
| 6 | 34 | 10 | | |
| 7 | 36 | 31 | 2 | 43 |
| 8 | 35 | | 1 | |
| 9 | 37 | 32 | | 44 |
| 10 | 38 | 33 | 4 | 45 |
| 11 | 1 | | | |
| 12 | 2 | | | |
| 13 | 4 | | | |
| 14 | 5 | | | |
| 1 | 1-8 | | | 1=15 to 40 |
| 1 | 1-2 | | | 1=41 to 48 |
| 49 | | 1 | | |
| 50 | | 2 | | |
| 51 | | 3 | | |
| 52 | | 5 | | |
| 53 | | 7 | | |
| 54 | | 9 | | |
| 1 | | 1-44 | | 1=55 to 74 |
| 1 | | 1-41 | | 1=75 to 80 |

Table 36. (Continued).

| College commodity number | Dept. A commodity numbers | Dept. B commodity numbers | Dept. "D" commodity numbers | Dept. E commodity numbers | |
|---|---|---|---|---|---|
| 81 | | | 3 | | |
| i | | | | i-81 | i=82 to 118 |
| i | | | | i-73 | i=119 to 130 |

Table 37. Cross reference college activity numbers (version two) and department activity numbers.

| College activity number | Dept. A activity number | Dept. B activity number | Dept. "D" activity number | Dept. E activity number | i = |
|---|---|---|---|---|---|
| i | i | | | | 1 to 59 |
| i | | i-59 | | | 60 to 109 |
| | | | i-109 | | 110 to 113 |
| | | | | i-113 | 114 to 195 |

Table 38. Constraints, college model (version two).

| Commodity constraint[a] | Reason for constraint |
| --- | --- |
| $y_1 \leqq - 725$ | Fixed target |
| $y_2 \leqq - 400$ | " " |
| $y_3 \leqq - 160$ | " " |
| $y_4 \leqq - .75$ | " " |
| $y_5 \leqq - 1.25$ | " " |
| $y_6 \leqq - 2.75$ | " " |
| $y_7 \leqq 0$ | Intermediate product (accounting) constraint |
| $y_8 \leqq 27$ | Resource allocation granted by university president |
| $y_9 \leqq 19125$ | Same as above |
| $y_{10} \leqq 1{,}095{,}000$ | " " " |
| $y_{11} \leqq - 30$ | Fixed target constraints faced only by department A |
| $y_{12} \leqq - 1400$ | Same as above |
| $y_{13} \leqq - 85$ | " " " |
| $y_{14} \leqq - 100$ | " " " |
| $y_{15} \leqq - 2.0$ | " " " |
| $A_i x - y_i \leqq 0$, $i=16,17,18$ | Accounting constraints (final, intermediate and primary products) faced only by department A |
| $y_i \leqq 0$, $i=19$ to $38$ | Same as above |
| $A_i x - y_i \leqq 0$, $i=39,40$ | " " " |

[a] $y_i = A_i x$ for $i = 1, 2 \ldots 130$.

Table 38. (Continued).

| Commodity constraint[a] | Reason for constraint |
|---|---|
| $y_{41} \leqq 21{,}000$ | Primary product constraint faced only by department A |
| $y_{42} \leqq 70{,}000$ | Same as above |
| $y_{43} \leqq 40{,}000$ | " " " |
| $0 \leqq y_{44} \leqq 2$ | " " " |
| $0 \leqq y_{45} \leqq 4$ | " " " |
| $y_{46} = 3$ | " " " |
| $y_{47} = 4.5$ | " " " |
| $y_{48} = 5$ | " " " |
| $y_{49} \leqq -120$ | Fixed target constraints faced only by department B |
| $y_{50} \leqq -6900$ | Same as above |
| $y_{51} \leqq -4700$ | " " " |
| $y_{52} \leqq -900$ | " " " |
| $y_{53} \leqq -225$ | " " " |
| $y_{54} \leqq -1$ | " " " |
| $y_{55} \leqq -3$ | " " " |
| $A_i x - y_i \leqq 0$, i=56 to 59 | Accounting constraints (final, intermediate, and primary products) faced only by department B |
| $y_i \leqq 0$, i-60 to 72 | Same as above |
| $A_i x - y_i \leqq 0$, i=73,74 | " " " |
| $y_{75} \leqq 20{,}000$ | Resource constraints faced only by department B |
| $y_{76} \leqq 25{,}000$ | Same as above |

Table 38. (Continued).

| Commodity constraint[a] | Reason for constraint |
|---|---|
| $0 \leqq y_{77} \leqq 4$ | Resource constraints faced only by department B |
| $0 \leqq y_{78} \leqq 8$ | Same as above |
| $y_{79} = 16$ | " " " |
| $y_{80} = 17.5$ | " " " |
| $y_{81} \leqq 25$ | Resource constraint faced only by department "D" |
| $y_{82} \leqq -100$ | Fixed target constraints faced only by department E |
| $y_{83} \leqq -3000$ | Same as above |
| $y_{84} \leqq -700$ | " " " |
| $y_{85} \leqq -200$ | " " " |
| $y_{86} \leqq -300$ | " " " |
| $y_{87} \leqq -15$ | " " " |
| $y_{88} \leqq -40$ | " " " |
| $y_{89} \leqq -75$ | " " " |
| $y_{90} \leqq -4$ | " " " |
| $y_{91} \leqq -2$ | " " " |
| $y_{92} \leqq -8$ | " " " |
| $A_i x - y_i \leqq 0$, $i=93,...96$ | Accounting constraints (final and intermediate products) faced only by department E |
| $y_i \leqq 0$, $i=97$ to $118$ | Same as above |
| $y_{119} \leqq 7200$ | Resource constraints faced only by department E |
| $y_{120} \leqq 80{,}000$ | Same as above |

Table 38. (Continued).

| Commodity constraint[a] | Reason for constraint |
|---|---|
| $y_{121} \leqq 125{,}000$ | Resource constraints faced only by department E |
| $0 \leqq y_{122} \leqq 6$ | Same as above |
| $0 \leqq y_{123} \leqq 8$ | " " " |
| $0 \leqq y_{124} \leqq 3$ | " " " |
| $0 \leqq y_{125} \leqq 4$ | Same as above |
| $y_{126} = 5$ | " " " |
| $y_{127} = 2$ | " " " |
| $y_{128} = 6.5$ | " " " |
| $y_{129} = 8.0$ | " " " |
| $y_{130} = 3.0$ | " " " |

Table 39. Objective function weights associated with flexible targets, college model (version two).

| Symbols used for flexible target levels[a] | Objective function weights |
|---|---|
| $y_{c16}$ $(-\ x_1)$ | - 2 |
| $y_{17}$ $(-\ x_2)$ | - 5 |
| $y_{18}$ $(-\ x_3)$ | - 4 |
| $x_{19}$ | 2.5 |
| $x_{20}$ | 4.9 |
| $y_{39}$ $(-\ x_{58})$ | 0.12 |
| $y_{40}$ $(-\ x_{59})$ | 1.5 |
| $y_{56}$ $(-\ x_{60})$ | - 1.5 |
| $y_{57}$ $(-\ x_{61})$ | - 2.25 |
| $y_{58}$ $(-\ x_{62})$ | - 2.5 |
| $y_{59}$ $(-\ x_{63})$ | - 2.0 |
| $x_{73}$ | 2.45 |
| $x_{74}$ | 4.75 |
| $y_{73}$ $(-\ x_{108})$ | 0.12 |
| $y_{74}$ $(-\ x_{109})$ | 1.5 |
| $y_{93}$ $(-\ x_{114})$ | - 2 |
| $y_{94}$ $(-\ x_{115})$ | - 2 |
| $y_{95}$ $(-\ x_{116})$ | - 4 |
| $y_{96}$ $(-\ x_{117})$ | - 4 |
| $x_{133}$ | 2.4 |

[a]The symbols shown in the parentheses were used for computing. They allowed the use of non-negative activity levels and positive objective function weights.

Table 39. (Continued).

| Symbols used for flexible target levels[a] | Objective function weights |
|---|---|
| $x_{134}$ | 2.4 |
| $x_{135}$ | 4.8 |
| $x_{136}$ | 4.8 |

Table 40. Solution to the dual of the college dean's problem.

| | Phase number | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
| $10v_1$ | 1.0 | .3 | .168 | 0 | .001 | 0 | .10 | .02 | .85 | 1.00 | .65 |
| $10v_2$ | 1.0 | .35 | .35 | .35 | .015 | .012 | .12 | 1.60 | .77 | .50 | .60 |
| $10v_3$ | 1.0 | 1.0 | .7 | 0 | .017 | .08 | 1.60 | 1.97 | 1.95 | .99 | 1.21 |
| $v_4$ | 1.0 | 1.0 | 1.0 | 0 | 0 | .14 | 0 | 0 | 3.37 | 3.84 | 1.94 |
| $v_5$ | 1.0 | 1.0 | 1.0 | 1.0 | .039 | 0 | 0 | 4.48 | 2.07 | 1.22 | 1.50 |
| $v_6$ | 1.0 | 1.0 | 1.0 | 0 | .020 | .08 | 1.95 | 2.72 | 2.66 | 1.22 | 1.50 |
| $v_7$ | | | | .037 | .017 | 0 | .09 | 5.27 | .04 | 3.13 | .21 |
| $10v_8$ | | | | | .015 | .06 | .61 | .13 | .26 | .37 | .37 |
| $100v_9$ | | | | | | 0 | 0 | 0 | 0 | 1.02 | 1.83 |
| $1000v_{10}$ | | | | | | .25 | 2.44 | .85 | .74 | 1.05 | 1.06 |
| $q_1$ | -1.0 | -1.0 | -1.0 | -.727 | -.59 | -22.37 | -364.35 | -119.23 | 19.82 | -32.76 | -73.90 |
| $q_2$ | -1.0 | -1.0 | -1.0 | 9.31 | -1.0 | -111.63 | -1449.87 | -357.05 | -337.83 | -657.71 | -726.34 |
| $q_3$ | -1.0 | -1.0 | 0 | 0 | 2.73 | 11.92 | 114.16 | 23.76 | 54.44 | 76.51 | 77.41 |
| $q_4$ | -1.0 | -1.0 | -.535 | -1.0 | -1.0 | -77.30 | -832.53 | -292.31 | -133.59 | -372.06 | -349.64 |

Table 40. (Continued).

| | Phase number 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
|---|---|---|---|---|---|---|---|---|---|
| $10v_1$ | .62 | .49 | .44 | .50 | .45 | .47 | .46 | .41 | .46 |
| $10v_2$ | .59 | .62 | .62 | .62 | .62 | .62 | .62 | .62 | .62 |
| $10v_3$ | 1.17 | 1.23 | 1.24 | 1.23 | 1.23 | 1.24 | 1.24 | 1.24 | 1.24 |
| $v_4$ | 1.85 | 1.63 | 1.49 | 1.68 | 1.50 | 1.56 | 1.54 | 1.37 | 1.52 |
| $v_5$ | 1.46 | 1.57 | 1.61 | 1.58 | 1.59 | 1.59 | 1.59 | 1.61 | 1.59 |
| $v_6$ | 1.46 | 1.57 | 1.60 | 1.58 | 1.59 | 1.59 | 1.59 | 1.61 | 1.59 |
| $v_7$ | 2.11 | .26 | .35 | .30 | .23 | .27 | .25 | .25 | .25 |
| $10v_8$ | .43 | .33 | .37 | .35 | .34 | .37 | .36 | .34 | .35 |
| $100v_9$ | .38 | 1.33 | 0 | .64 | .53 | .23 | .21 | .08 | .38 |
| $1000v_{10}$ | 1.22 | .95 | 1.06 | .99 | .98 | 1.06 | 1.04 | 0.97 | 1.00 |
| $q_1$ | -52.90 | -49.95 | -27.41 | -31.72 | -31.08 | -30.66 | -28.62 | -19.36 | -29.50 |
| $q_2$ | -704.30 | -609.03 | -561.89 | -575.14 | -599.19 | -581.16 | -570.27 | -513.63 | -563.21 |
| $q_3$ | 89.13 | 69.48 | 77.38 | 72.45 | 71.80 | 77.22 | 76.23 | 71.16 | 73.78 |
| $q_4$ | -354.36 | -271.60 | -218.51 | -239.15 | -224.79 | -232.19 | -225.05 | -189.46 | -224.35 |

Table 40. (Continued).

| | Phase number 21 | 22 |
|---|---|---|
| $10v_1$ | .47 | .47 |
| $10v_2$ | .62 | .62 |
| $10v_3$ | 1.23 | 1.23 |
| $v_4$ | 1.57 | 1.57 |
| $v_5$ | 1.58 | 1.58 |
| $v_6$ | 1.58 | 1.58 |
| $v_7$ | .25 | .25 |
| $10v_8$ | .36 | .36 |
| $100v_9$ | .45 | .47 |
| $1000v_{10}$ | 1.02 | 1.02 |
| $q_1$ | -32.60 | -32.67 |
| $q_2$ | -579.13 | -578.52 |
| $q_3$ | 74.80 | 74.59 |
| $q_4$ | -235.15 | -235.07 |

Table 41. Quotas requested by department A.

| | Phase number | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | 1[a] | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| $\gamma_a$ | 0 | 14.85 | 9.39 | 38.89 | 10.61 | 153.47 | 74.0 | 81.82 | 102.93 | 93.53 |
| $1/10u_{a1}$ | | -500.0 | 15.0 | -500.0 | 15.0 | 17.66 | 15.47 | 15.53 | 15.49 | 15.33 |
| $u_{a3}$ | | 1.34 | .84 | 3.45 | .95 | 9.53 | 2.84 | 2.95 | 5.20 | 4.37 |
| $u_{a4}$ | 1.05 | 83.89 | -41.48 | 85.32 | -41.41 | -14.80 | -7.58 | -11.93 | -38.06 | -38.81 |
| $u_{a6}$ | | .411 | .26 | 1.06 | .294 | 2.93 | .88 | .91 | 1.60 | 1.35 |
| $u_{a7}$ | | 19.75 | 19.75 | 0 | 0 | 19.75 | 19.75 | 0 | 19.75 | 0 |
| $1/10u_{a8}$ | .3 | 59.09 | 14.74 | 68.20 | 22.69 | 19.71 | 15.82 | 23.79 | 16.85 | 24.32 |
| $1/100u_{a9}$ | .028 | 32.26 | 25.06 | 30.33 | 23.86 | 53.18 | 25.53 | 26.61 | 34.50 | 33.20 |
| $1/1000u_{a10}$ | .068 | 174.42 | 147.66 | 174.66 | 146.81 | 273.88 | 93.40 | 100.35 | 149.35 | 146.84 |

[a]Based on a homogeneous solution.

Table 41. (Continued).

| | Phase number | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| $\gamma_a$ | 83.03 | 87.73 | 83.10 | 89.80 | 103.79 | 91.93 | 92.84 | 106.81 | 109.34 | 107.73 |
| $1/10u_{a1}$ | -500.0 | -500.0 | 15.53 | 15.55 | -500.0 | -500.0 | -500.0 | -500.0 | -500.0 | -500.0 |
| $u_{a3}$ | 3.26 | 3.56 | 3.24 | 3.78 | 5.25 | 3.93 | 4.01 | 5.51 | 5.77 | 5.59 |
| $u_{a4}$ | 94.64 | 101.96 | -30.32 | -26.23 | 87.14 | 103.11 | 103.57 | 88.54 | 89.79 | 89.00 |
| $u_{a6}$ | 1.00 | 1.10 | 1.00 | 1.16 | 1.62 | 1.21 | 1.23 | 1.70 | 1.77 | 1.72 |
| $u_{a7}$ | 19.75 | 0 | 0 | 0 | 0 | 19.75 | 0 | 19.75 | 0 | 0 |
| $1/10u_{a8}$ | 60.08 | 68.14 | 23.92 | 24.16 | 68.90 | 60.49 | 68.45 | 61.18 | 69.22 | 69.14 |
| $1/100u_{a9}$ | 35.81 | 36.29 | 29.00 | 30.23 | 41.63 | 36.32 | 36.59 | 41.47 | 41.97 | 41.47 |
| $1/1000u_{a10}$ | 151.14 | 144.93 | 126.95 | 124.69 | 177.82 | 144.27 | 146.31 | 176.05 | 178.20 | 178.09 |

Table 42. Quotas requested by department B.

| | Phase number | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | 1[a] | 2[a] | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
| $\gamma_b$ | | | 48.47 | 44.63 | 43.91 | 138.77 | 55.25 | 66.24 | 66.24 | 104.42 | 53.84 |
| $1/10u_{b2}$ | -3.0 | | 72.0 | 72.0 | 72.0 | 72.0 | 72.0 | 72.0 | 72.0 | 72.0 | 72.0 |
| $1/10u_{b3}$ | | -1.5 | 9.07 | 9.93 | 8.22 | 28.93 | 9.98 | 11.38 | 11.38 | 19.67 | 8.81 |
| $u_{b5}$ | 1.05 | | -34.51 | -48.0 | -48.0 | 3.31 | 4.80 | -57.48 | -12.82 | -27.20 | -27.20 |
| $u_{b6}$ | | 1.05 | -36.38 | -8.75 | -9.96 | -42.08 | -57.0 | -12.68 | -57.33 | -1.65 | -9.88 |
| $1/10u_{b8}$ | .3 | .3 | 125.63 | 122.77 | 122.77 | 128.41 | 121.82 | 125.81 | 125.81 | 122.65 | 118.46 |
| $1/100u_{b9}$ | .028 | .028 | 88.77 | 82.55 | 85.47 | 96.88 | 81.84 | 86.56 | 86.56 | 82.52 | 78.47 |
| $1/1000u_{b10}$ | .068 | .068 | 603.51 | 580. | 587.01 | 661.20 | 576.14 | 600.23 | 600.23 | 576.31 | 558.66 |

[a]Based on homogeneous solutions.

Table 42. (Continued).

| | Phase number 12 | 13 | 14 | 15 |
|---|---|---|---|---|
| $\gamma_b$ | 101.87 | 65.72 | 65.71 | 65.72 |
| $1/10u_{b2}$ | 72.0 | 72.0 | 72.0 | 72.00 |
| $1/10u_{b3}$ | 18.15 | 11.38 | 11.38 | 11.38 |
| $u_{b5}$ | -44.15 | -33.22 | -57.48 | -56.55 |
| $u_{b6}$ | -10.48 | -36.93 | -12.68 | -13.61 |
| $1/10u_{b8}$ | 126.97 | 125.81 | 125.81 | 125.81 |
| $1/100u_{b9}$ | 87.28 | 86.32 | 86.32 | 86.32 |
| $1/1000u_{b10}$ | 601.97 | 599.63 | 599.64 | 599.64 |

Table 43. Quotas requested by department D.

| | 1 | 5 | 6[a] | 7 |
|---|---|---|---|---|
| $u_{d7}$ | | | 10.0 | |
| $1/10u_{d8}$ | -187.5 | 262.5 | -1.0 | -225.0 |
| $1/1000u_{d10}$ | | 18.75 | | 5.625 |

[a]Based on a homogeneous solution.

Table 44. Quotas requested by department E.

| | Phase number | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | 2 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| $\gamma_e$ | 10.54 | 22.59 | 12.13 | 221.15 | 96.09 | 217.90 | 198.30 | 136.60 | 135.23 | 135.23 |
| $1/10u_{e1}$ | .88 | 2.35 | 1.26 | 15.83 | 2.42 | 15.85 | 13.73 | 5.54 | 6.26 | 6.26 |
| $1/10u_{e2}$ | .44 | .94 | .51 | 6.62 | .87 | 6.46 | 5.59 | 2.72 | 2.52 | 2.58 |
| $u_{e4}$ | .16 | .40 | .22 | 2.59 | .35 | 2.61 | 2.19 | 1.03 | .99 | 1.03 |
| $u_{e5}$ | .07 | .10 | .05 | .93 | .11 | .84 | .79 | .42 | .36 | 0.35 |
| $u_{e7}$ | 21.08 | 0 | 17.06 | 26.4 | 26.4 | 0 | 26.4 | 0 | 26.4 | 26.4 |
| $1/10u_{e8}$ | 42.97 | 67.32 | 46.59 | 54.86 | 45.61 | 82.41 | 51.40 | 77.87 | 47.48 | 47.48 |
| $1/100u_{e9}$ | 43.34 | 52.25 | 44.58 | 84.62 | 64.15 | 86.82 | 71.44 | 77.53 | 55.44 | 66.89 |
| $1/1000u_{10}$ | 334.57 | 375.72 | 340.14 | 416.97 | 285.89 | 421.62 | 353.83 | 360.64 | 302.85 | 303.69 |

Table 44. (Continued).

| | Phase number | | | | | | |
|---|---|---|---|---|---|---|---|
| | 13 | 14 | 15 | 16 | 17 | 18 | 21 |
| $\gamma_e$ | 150.75 | 195.76 | 180.87 | 197.57 | 165.48 | 195.16 | 170.45 |
| $1/10u_{e1}$ | 8.15 | 13.69 | 11.75 | 13.85 | 10.06 | 13.62 | 10.65 |
| $1/10u_{e2}$ | 3.48 | 5.43 | 4.81 | 5.55 | 3.97 | 5.40 | 4.21 |
| $u_{e4}$ | 1.37 | 2.16 | 1.88 | 2.19 | 1.58 | 2.14 | 1.67 |
| $u_{e5}$ | 0.49 | 0.74 | 0.69 | 0.77 | 0.54 | 0.73 | 0.58 |
| $u_{e7}$ | 26.4 | 26.4 | 26.4 | 26.4 | 26.4 | 26.4 | 26.4 |
| $1/10u_{e8}$ | 48.49 | 51.17 | 50.32 | 51.34 | 49.25 | 51.15 | 49.56 |
| $1/100u_{e9}$ | 64.51 | 7.69 | 68.61 | 70.64 | 68.68 | 70.96 | 69.06 |
| $1/1000u_{10}$ | 317.36 | 351.44 | 340.03 | 353.10 | 327.31 | 350.98 | 331.25 |

Table 45. An optimum set of quotas.

| | $u_{aj}$ (Dept. A) | $u_{bj}$ (Dept. B) | $u_{dj}$ (Dept. D) | $u_{ej}$ (Dept. E) |
|---|---|---|---|---|
| 1 | -860.85 | 0.0 | 0.0 | 135.85 |
| 2 | 0.0 | -453.83 | 0.0 | 53.83 |
| 3 | 40.63 | -200.62 | 0.0 | 0.0 |
| 4 | -2.89 | 0.0 | 0.0 | 2.14 |
| 5 | 0.0 | -1.98 | 0.0 | 0.73 |
| 6 | 1.25 | -4.00 | 0.0 | 0.0 |
| 7 | 327.56 | 1439.73 | -2278.61 | 511.33 |
| 8 | 0.6 | 0.0 | 0.0 | 26.4 |
| 9 | 3226.20 | 8805.89 | 0.0 | 7093.00 |
| 10 | 133,750.62 | 603,863.63 | 6,626.44 | 350,763.53 |

Table 46. Upper and lower bound estimate of the optimum value of the college dean's objective function.

| Phase | Lower bound estimate[a] | Upper bound estimate[b] |
|---|---|---|
| 6 | 76.96 | 438.79 |
| 7 | 94.02 | 438.79 |
| 8 | 213.23 | 438.79 |
| 9 | 282.34 | 405.39 |
| 10 | 323.28 | 402.31 |
| 11 | 345.20 | 402.31 |
| 12 | 346.45 | 402.31 |
| 13 | 352.97 | 358.62 |
| 14 | 353.23 | 355.65 |
| 15 | 354.52 | 355.65 |
| 16 | 354.79 | 355.65 |
| 17 | 355.00 | 355.64 |
| 18 | 355.02 | 355.64 |
| 19 | 355.13 | 355.64 |
| 20 | 355.15 | 355.32 |
| 21 | 355.16 | 355.19 |
| 22 | 355.17 | 355.17 |

[a]The lower bound estimate at the end of the kth phase was obtained by setting it equal to the value attained by the objective function of (the kth phase version of) 7.15.

[b]The upper bound estimate at the end of the kth phase was obtained by setting it equal to

$$\min_{6 \leq j \leq k} \quad (\hat{\bar{v}}\langle j\rangle)' \,\bar{b} + \sum_{i=1}^{n} \hat{z}_i \langle j\rangle \quad , \text{ where}$$

$$\hat{z}_i \langle j\rangle = \hat{\Omega}_i \langle j\rangle + \hat{q}_i \langle j\rangle \, .$$

Table 47. An optimal solution to the dual of 7.15 (obtained after classroom use constraints were removed.)

| Variable | Solution value |
|---|---|
| $10\bar{v}_1$ | 0.38 |
| $10\bar{v}_2$ | 0.60 |
| $10\bar{v}_3$ | 1.20 |
| $\bar{v}_4$ | 1.38 |
| $\bar{v}_5$ | 1.65 |
| $\bar{v}_6$ | 1.65 |
| $\bar{v}_7$ | 0.0 |
| $\bar{v}_8$ | 0.0 |
| $\bar{v}_9$ | 0.08 |
| $\bar{v}_{10}$ | 1.01 |
| $q_A$ | -12.45 |
| $q_B$ | -485.55 |
| $q_E$ | -176.10 |

Table 48. An optimal set of quotas (obtained after classroom use constraints were removed.)

| j | $u_{aj}$ | $u_{bj}$ | $u_{ej}$ |
|---|---|---|---|
| 1 | -862.75 | 0.0 | 137.75 |
| 2 | 0.0 | -454.99 | 54.99 |
| 3 | 41.57 | -201.56 | 0.0 |
| 4 | -2.93 | 0.0 | 2.18 |
| 5 | 0.0 | -2.01 | 0.76 |
| 6 | 1.28 | -4.03 | 0.0 |
| 7 | 19.75 | 0.0 | 26.40 |
| 8 | 251.18 | 1439.84 | 512.70 |
| 9 | 3228.43 | 8805.33 | 7091.25 |
| 10 | 132,531.51 | 603,849.96 | 352,354.69 |

# XV. APPENDIX D

Table 49. Space allocation "department" activities.

| | Activity numbers | $d_1$ | $d_2$ | $d_3$ | $d_4$ |
|---|---|---|---|---|---|
| | Commodity numbers | | | | |
| Small classroom units | $d_1$ | -75 | -90 | -105 | -1 |
| Large classroom units | $d_2$ | | | | 1 |
| Small classrooms | $d_3$ | 1 | 1 | 1 | |
| Teaching budget funds | $d_4$ | | 225 | 750 | |

Table 50. Solution to department D model after constraints are imposed.

| $x_d$ subscript | Solution value |
|---|---|
| 1 | 0.0 |
| 2 | 23.09 |
| 3 | 1.91 |
| 4 | 0.0 |

Table 51. Solution to the dual of the department D model.

| Variable | Solution value[a] | |
|---|---|---|
| $v_{d1}$ | 0.0 | (0.036) |
| $v_{d2}$ | 0.0 | (0.252) |
| $v_{d3}$ | 0.0 | (2.983) |
| $1000v_{d4}$ | 0.0 | (1.020) |

# XVI. APPENDIX E

Table 52. Final products, Economics department.

| Commodity number | Product description | Unit of measurement |
|---|---|---|
| $e_o$[a] | Ph.D. degrees (specialty one) awarded by the Economics department. | One degree per academic year. |
| $e_{oo}$[a] | Ph.D. degrees (specialty two) awarded by the Economics department. | Same as above. |
| $e_{ooo}$[a] | Masters' degrees (specialty one) awarded by the Economics department. | Same as above. |
| $e_{oooo}$[a] | Masters' degrees (specialty two) awarded by the Economics department. | Same as above. |
| $e_1$ | Training of undergraduate Economics majors. | One undergraduate unit per year. One unit is the amount of instruction (and departmental administration) in Economics which is required to produce one Bachelors' (in Economics) degree per year. |
| $e_2$ | Instruction in Economics Principles. | One instructional unit per year. An instructional unit is one (3 quarter hours) course taught to one student. |
| $e_3$ | Undergraduate instruction (specialty one). | Same as above. |

[a]Commodities $e_o$ through $e_{oooo}$ are commodities for which no constraints have been included in the model.

Table 52. (Continued).

| Commodity number | Product description | Unit of measurement |
|---|---|---|
| $e_4$ | Undergraduate instruction (specialty two). | One instructional unit per year. An instructional unit is one (3 quarter hours) course taught to one student. |
| $e_5$ | Undergraduate instruction (specialty three). | Same as above. |
| $e_6$ | Graduate instruction (specialty one). | " " " |
| $e_7$ | Graduate instruction (specialty two). | " " " |
| $e_8$ | Graduate instruction (specialty three). | " " " |
| $e_9$ | Faculty teaching services (graduate teaching, specialty one). | One unit of teaching per year. A unit of teaching is the amount required to teach (including preparation) one (3 quarter hours) course. |
| $e_{10}$ | Faculty teaching services (graduate teaching, specialty two). | Same as above. |
| $e_{11}$ | Faculty teaching services (graduate teaching, specialty three). | " " " |
| $e_{12}$ | Publications (non-project research) by "specialty one" faculty members. | One publication (25 pages) per year. |

Table 52. (Continued).

| Commodity number | Product description | Unit of measurement |
|---|---|---|
| $e_{13}$ | Publications (project research) by "specialty one" faculty members. | One publication (25 pages) per year. |
| $e_{14}$ | Faculty publications in specialty area two. | Same as above. |
| $e_{15}$ | Faculty publications in specialty area three. | " " " |

Table 53. Intermediate products, Economics department.

| Commodity number | Product description | Unit of measurement |
|---|---|---|
| $e_{16}$ | Faculty teaching services (undergraduate teaching, principles). | Same as for $e_9$. |
| $e_{17}$ | Faculty teaching services (undergraduate teaching, specialty one). | Same as for $e_9$. |
| $e_{18}$ | Faculty teaching services (undergraduate teaching, specialty two). | Same as for $e_9$. |
| $e_{19}$ | Faculty teaching services (undergraduate teaching, specialty three). | Same as for $e_9$. |
| $e_{20}$ | Teaching services provided by graduate instructors. | Same as for $e_9$. |
| $e_{21}$ | Teaching services provided by graduate teaching assistants. | One unit of teaching assistance per year. One unit is the amount of assistance required to "teach" one recitation section per week for one quarter. |
| $e_{22}$ | Secretarial and clerical assistance used by teaching activities. | One woman year (9-10 months) per year. |
| $e_{23}$ | Research assistance (specialty two research) provided by Masters' candidates. | One half (4.5 months full time equivalent) man year per year. |
| $e_{24}$ | Research assistance (specialty three research) provided by Masters' candidates. | Same as above. |
| $e_{25}$ | Research assistance (specialty two research) provided by Ph.D. candidates. | Same as above. |

Table 53. (Continued).

| Commodity numbers | Product description | Unit of measurement |
|---|---|---|
| $e_{26}$ | Research assistance (specialty three research) provided by Ph.D. candidates. | One half (4.5 months full time equivalent) man year per year. |
| $e_{27}$ | Secretarial and clerical assistance used by "specialty one" project research. | One woman year (9-10 months) per year. |
| $e_{28}$ | Secretarial and clerical assistance used by "specialty two" research. | Same as above. |
| $e_{29}$ | Secretarial and clerical assistance used by "specialty three" research. | " " " |
| $e_{30}$ | Faculty research services (specialty one, non-project research). | One man year (9-10 months) per year. |
| $e_{31}$ | Faculty research services (specialty one, project research). | Same as above. |
| $e_{32}$ | Faculty research services (specialty two research). | " " " |
| $e_{33}$ | Faculty research services (specialty three research). | " " " |
| $e_{34}$ | Counselling and advising of undergraduate students. | One unit of counselling per year. One unit is the amount of advising required per student (discipline A major) per quarter. |
| $e_{35}$ | Counselling and advising of graduate students. | Same as above. |

Table 53. (Continued).

| Commodity number | Product description | Unit of measurement |
|---|---|---|
| $e_{36}$ | Thesis (and related) supervision of "specialty two" graduate students. | The supervision of 3 thesis credit hours per year. |
| $e_{37}$ | Thesis (and related) supervision of "specialty three" graduate students. | Same as above. |

Table 54. Primary products, Economics department.

| Commodity number | Product description | Unit of measurement |
|---|---|---|
| $e_{38}$ | Instruction provided for Economics department graduate students by department A. | Same as for $e_2$. |
| $e_{39}$ | Instruction provided for Economics department graduate students by department B. | Same as for $e_2$. |
| $e_{40}$ | Teaching services provided to the Economics department by department A. | Same as for $e_2$. |
| $e_{41}$ | Teaching services provided to the Economics department by department B. | Same as for $e_2$. |
| $e_{42}$ | Large classroom use. | One classroom unit per year. One unit is the amount (number of hours) of classroom use required to conduct a class meeting one hour per week for one quarter. |
| $e_{43}$ | Small classroom use. | Same as above. |
| $e_{44}$ | Office space use. | One square foot. |
| $e_{45}$ | Teaching budget funds. | One dollar per year. |
| $e_{46}$ | "Specialty one" (project) research funds. | Same as above. |
| $e_{47}$ | "Specialty two" research funds. | Same as above. |

Table 54. (Continued).

| Commodity number | Product description | Unit of measurement |
|---|---|---|
| $e_{48}$ | "Specialty three" research funds. | One dollar per year. |
| $e_{49}$ | "Specialty two" Masters' candidate. | One new candidate per year. |
| $e_{50}$ | "Specialty three" Masters' candidate. | Same as above. |
| $e_{51}$ | "Specialty two" Ph.D. candidate. | Same as above. |
| $e_{52}$ | "Specialty three" Ph.D. candidates. | Same as above. |
| $e_{53}$ | "Specialty one" faculty members who enjoy no outside (non-university) research support. | One faculty member. |
| $e_{54}$ | "Specialty one" faculty members who enjoy outside (non-university) research support. | Same as above. |
| $e_{55}$ | "Specialty two" faculty members. | Same as above. |
| $e_{56}$ | "Specialty three" faculty members. | Same as above. |
| $e_{57}$ | Faculty members proficient in both "Specialty one" and "Specialty three". | Same as above. |

Table 55. Research activities.

| Activity numbers | $e_5$ | $e_6$ | $e_7$ | $e_8$ | $e_9$ | $e_{10}$ | $e_{11}$ | $e_{12}$ | $e_{13}$ | $e_{14}$ | $e_{15}$ | $e_{16}$ | $e_{17}$ | $e_{18}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Commodity numbers | | | | | | | | | | | | | | |
| $e_{12}$ | -1.0 | -1.1 | | | | | | | | | | | | |
| $e_{13}$ | | | -1.1 | -1.25 | | | | | | | | | | |
| $e_{14}$ | | | | | -3.0 | -2.25 | -3.5 | -2.0 | -1.6 | | | | | |
| $e_{15}$ | | | | | | | | | | -3.0 | -2.7 | -3.1 | -2.1 | -1.6 |
| $e_{22}$ | .1 | .3 | | | | | | | | | | | | |
| $e_{23}$ | | | | | 2.0 | 4.0 | | | | | | | | |
| $e_{24}$ | | | | | | | | | | 2.0 | 6.0 | | | |
| $e_{25}$ | | | | | 4.0 | | 8.0 | | | | | | | |
| $e_{26}$ | | | | | | | | | | 4.0 | | 6.0 | | |
| $e_{27}$ | | | .1 | .3 | | | | | | | | | | |
| $e_{28}$ | | | | | .75 | .5 | 1.0 | 1.5 | .5 | | | | | |
| $e_{29}$ | | | | | | | | | | 1.0 | .75 | 1.25 | 1.5 | .5 |
| $e_{30}$ | 1 | 1 | | | | | | | | | | | | |
| $e_{31}$ | | | 1 | 1 | | | | | | | | | | |
| $e_{32}$ | | | | | 1 | 1 | 1 | 1 | 1 | | | | | |
| $e_{33}$ | | | | | | | | | | 1 | 1 | 1 | 1 | 1 |
| $e_{46}$ | | | 800 | 500 | | | | | | | | | | |
| $e_{47}$ | | | | | 4500 | 2500 | 5000 | 2000 | 1000 | | | | | |
| $e_{48}$ | | | | | | | | | | 4500 | 3500 | 4500 | 3000 | 1000 |

Table 56. "Degree" activities.

| Activity numbers | $e_{19}$ | $e_{20}$ | $e_{21}$ | $e_{22}$ | $e_{23}$ |
|---|---|---|---|---|---|
| Commodity numbers | | | | | |
| $e_1$ | -1 | | | | |
| $e_2$ | 3 | | | | |
| $e_3$ | 2 | | | | |
| $e_4$ | 4 | | | | |
| $e_5$ | 3 | | | | |
| $e_6$ | | 5 | 6 | 6 | 7 |
| $e_7$ | | 3 | 1 | 5 | 2 |
| $e_8$ | | 1 | 2 | 1 | 4 |
| $e_{34}$ | 13 | | | | |
| $e_{35}$ | | 6 | 6 | 10 | 10 |
| $e_{36}$ | | 4 | | 8 | |
| $e_{37}$ | | | 4 | | 8 |
| $e_{38}$ | | 3 | 3 | 5 | 4 |
| $e_{39}$ | | 1 | 1 | 2 | 2 |
| $e_{49}$ | | 1 | | | |
| $e_{50}$ | | | 1 | | |
| $e_{51}$ | | | | 1 | |
| $e_{52}$ | | | | | 1 |

Table 57. Undergraduate teaching and advising activities.

| Activity numbers | Advising | Economics principles teaching | | | | | | Undergraduate teaching | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $e_{24}$ | $e_{25}$ | $e_{26}$ | $e_{27}$ | $e_{28}$ | $e_{29}$ | $e_{30}$ | $e_{31}$ | $e_{32}$ | $e_{33}$ | $e_{34}$ | $e_{35}$ | $e_{36}$ |
| Commodity numbers | | | | | | | | | | | | | |
| $e_2$ | | -20.0 | -60.0 | -250.0 | -400.0 | -290.0 | -35.0 | | | | | | |
| $e_3$ | | | | | | | | -25.0 | -30.0 | | | | |
| $e_4$ | | | | | | | | | | -25.0 | -30.0 | | |
| $e_5$ | | | | | | | | | | | | -25.0 | -30.0 |
| $e_{16}$ | 1.0 | .1 | 1.25 | 1.4 | 5.0 | 3.45 | 1.1 | | | | | | |
| $e_{17}$ | | | | | | | | .6 | 1.1 | | | | |
| $e_{18}$ | | | | | | | | | | .85 | 1.1 | | |
| $e_{19}$ | | | | | | | | | | | | .85 | 1.1 |
| $e_{20}$ | | 1.0 | | 1.5 | | | | .5 | | .25 | | .25 | |
| $e_{21}$ | | | 2.0 | 15.0 | 21.0 | 15.0 | | | | | | | |
| $e_{22}$ | .1 | .02 | .03 | .07 | .13 | .08 | .02 | .02 | .02 | .02 | .02 | .02 | .02 |
| $e_{34}$ | -50.0 | | | | | | | | | | | | |
| $e_{42}$ | | | 2 | 2 | 8 | 2 | | | | | | | |
| $e_{43}$ | | 3 | 3 | 14.5 | 19.0 | 16.0 | 3.0 | 3.0 | 3.0 | 3.0 | 3.0 | 3.0 | 3.0 |

Table 58. Graduate teaching, advising and thesis supervision.

| Activity numbers | Graduate advising $e_{37}$ | Thesis supervision $e_{38}$ | $e_{39}$ | Graduate teaching $e_{40}$ | $e_{41}$ | $e_{42}$ |
|---|---|---|---|---|---|---|
| Commodity numbers | | | | | | |
| $e_6$ | | | | -20.0 | | |
| $e_7$ | | | | | -15.0 | |
| $e_8$ | | | | | | -15.0 |
| $e_9$ | 1.0 | .2 | .20 | 1.05 | | |
| $e_{10}$ | | .65 | .10 | | 1.05 | |
| $e_{11}$ | | .10 | .65 | | | 1.05 |
| $e_{22}$ | .4 | .02 | .02 | .02 | .02 | .02 |
| $e_{35}$ | -100.0 | | | | | |
| $e_{36}$ | | -7.5 | | | | |
| $e_{37}$ | | | -7.5 | | | |
| $e_{40}$ | | .08 | .07 | | | |
| $e_{41}$ | | .02 | .03 | | | |
| $e_{43}$ | | | | 3 | 3 | 3 |

Table 59. Graduate student appointments.

| Activity numbers | $e_{43}$ | Masters candidate appointments $e_{44}$ | $e_{45}$ | $e_{46}$ | $e_{47}$ | $e_{48}$ | $e_{49}$ | Ph.D. candiate appointments $e_{50}$ | $e_{51}$ | $e_{52}$ | $e_{53}$ | $e_{54}$ | $e_{55}$ | $e_{56}$ | $e_{57}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Commodity numbers | | | | | | | | | | | | | | | |
| $e_{20}$ | 1 | | | | | | -3.6 | -3.0 | -6.0 | -5.5 | | -3.6 | -3.0 | -6.0 | -5.5 |
| $e_{21}$ | -3 | | -18 | | -18 | | | -1.8 | | -1.5 | | | -1.8 | | -1.5 |
| $e_{23}$ | | -1 | | | | | | | | | | | | | |
| $e_{24}$ | | | | -1 | | | | | | | | | | | |
| $e_{25}$ | | | | | | -1 | -.4 | -.4 | | | | | | | |
| $e_{26}$ | | | | | | | | | | | -1 | -.4 | -.4 | | |
| $e_{44}$ | | 40 | 40 | 40 | 40 | 47 | 60 | 60 | 60 | 60 | 47 | 60 | 60 | 60 | 60 |
| $e_{45}$ | | | 2850 | | 2850 | | 2040 | 1985 | 3400 | 3355 | | 2040 | 1985 | 3400 | 3355 |
| $e_{47}$ | | 2750 | | | | 3000 | 1270 | 1270 | | | | | | | |
| $e_{48}$ | | | | 2750 | | | | | | | 3000 | 1270 | 1270 | | |
| $e_{49}$ | | -.5 | -.5 | | | | | | | | | | | | |
| $e_{50}$ | | | | -.5 | -.5 | | | | | | | | | | |
| $e_{51}$ | | | | | | -1/3 | -3/10 | -3/10 | -1/4 | -1/4 | | | | | |
| $e_{52}$ | | | | | | | | | | | -1/3 | -3/10 | -3/10 | -1/4 | -1/4 |

Table 60. Secretarial activities.

| Activity number | $e_{58}$ | $e_{59}$ | $e_{60}$ | $e_{61}$ |
|---|---|---|---|---|
| Commodity number | | | | |
| $e_{22}$ | -1 | | | |
| $e_{27}$ | | -1 | | |
| $e_{28}$ | | | -1 | |
| $e_{29}$ | | | | -1 |
| $e_{44}$ | 140 | 140 | 140 | 140 |
| $e_{45}$ | 3400 | | | |
| $e_{46}$ | | 3400 | | |
| $e_{47}$ | | | 3400 | |
| $e_{48}$ | | | | 3400 |

Table 61. Faculty allocation activities.

| Activity numbers | "Transfer" activities $e_{62}$ | $e_{63}$ | $e_{64}$ | $e_{65}$ | "Specialty one" faculty allocations $e_{66}$ | $e_{67}$ | $e_{68}$ | $e_{69}$ | $e_{70}$ | $e_{71}$ |
|---|---|---|---|---|---|---|---|---|---|---|
| Commodity numbers | | | | | | | | | | |
| $e_9$ | | | | | -2.0 | -5.0 | | -2.0 | -5.0 | |
| $e_{10}$ | | | | | | | | | | |
| $e_{11}$ | | | | | | | | | | |
| $e_{16}$ | -1.0 | -1.0 | -1.0 | | | | | | | |
| $e_{17}$ | 1.0 | | | | -6.0 | -3.0 | | -6.0 | -3.0 | |
| $e_{18}$ | | 1.0 | | | | | | | | |
| $e_{19}$ | | | 1.0 | | | | | | | |
| $e_{30}$ | | | | -1 | -.25 | -.25 | -1.0 | | | |
| $e_{31}$ | | | | 1 | | | | -.25 | -.25 | -1.0 |
| $e_{32}$ | | | | | | | | | | |
| $e_{33}$ | | | | | | | | | | |
| $e_{44}$ | | | | | 125 | 125 | 125 | 125 | 125 | 125 |
| $e_{45}$ | | | | 13000 | 12250 | 12250 | 12250 | 9750 | 9750 | |
| $e_{46}$ | | | | -13000 | | | | 3250 | 3250 | 13000 |
| $e_{47}$ | | | | | | | | | | |
| $e_{48}$ | | | | | | | | | | |
| $e_{53}$ | | | | | 1 | 1 | 1 | | | |
| $e_{54}$ | | | | | | | | 1 | 1 | 1 |
| $e_{55}$ | | | | | | | | | | |
| $e_{56}$ | | | | | | | | | | |

Table 61. (Continued).

| Activity numbers | "Specialty two" faculty allocations $e_{72}$ | $e_{73}$ | $e_{74}$ | "Specialty three" faculty allocations $e_{75}$ | $e_{76}$ | $e_{77}$ | Allocation activities for faculty members proficient in both "Specialties" "one" and "three" $e_{78}$ | $e_{79}$ | $e_{80}$ | $e_{81}$ | $e_{82}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Commodity numbers | | | | | | | | | | | |
| $e_9$ | | | | | | | -2.0 | -5.0 | | | |
| $e_{10}$ | -2.0 | -5.0 | | | | | | | | | |
| $e_{11}$ | | | | -2.0 | -5.0 | | | | -2.0 | -5.0 | |
| $e_{16}$ | | | | | | | | | | | |
| $e_{17}$ | | | | | | | | | | | |
| $e_{18}$ | -6.0 | -3.0 | | | | | | | | | |
| $e_{19}$ | | | | -6.0 | -3.0 | | -6.0 | -3.0 | -6.0 | -3.0 | |
| $e_{30}$ | | | | | | | | | | | |
| $e_{31}$ | | | | | | | | | | | |
| $e_{32}$ | -.25 | -.25 | -1.0 | | | | | | | | |
| $e_{33}$ | | | | -.25 | -.25 | -1.0 | -.25 | -.25 | -.25 | -.25 | -1.0 |
| $e_{44}$ | 125 | 125 | 125 | 125 | 125 | 125 | 125 | 125 | 125 | 125 | 125 |
| $e_{45}$ | 12375 | 12375 | | 11250 | 11250 | | 11812 | 11812 | 11812 | 11812 | |
| $e_{46}$ | | | | | | | | | | | |
| $e_{47}$ | 4125 | 4125 | 16500 | | | | | | | | |
| $e_{48}$ | | | | 3750 | 3750 | 15000 | 3938 | 3938 | 3938 | 3938 | 15750 |
| $e_{54}$ | | | | | | | | | | | |
| $e_{55}$ | 1 | 1 | 1 | | | | | | | | |
| $e_{56}$ | | | | 1 | 1 | 1 | | | | | |
| $e_{57}$ | | | | | | | 1 | 1 | 1 | 1 | 1 |

Table 62. Constraints, Economics department model.

| Commodity constraint[a] | Reason for constraint |
|---|---|
| $y_{e1} \leqq -100.0$ | Fixed target |
| $y_{e2} \leqq -3000.0$ | " " |
| $y_{e3} \leqq -700.0$ | " " |
| $y_{e4} \leqq -200.0$ | " " |
| $y_{e5} \leqq -300.0$ | " " |
| $y_{e6} \leqq -15.0$ | " " |
| $y_{e7} \leqq -40.0$ | " " |
| $y_{e8} \leqq -75.0$ | " " |
| $y_{e9} \leqq -4$ | " " |
| $y_{e10} \leqq -2$ | " " |
| $y_{e11} \leqq -8$ | " " |
| $A_{e12}x_e - y_{e12} \leqq 0$ | Accounting constraint |
| $A_{e13}x_e - y_{e13} \leqq 0$ | " " |
| $A_{e14}x_e - y_{e14} \leqq 0$ | " " |
| $A_{e15}x_e - y_{e15} \leqq 0$ | " " |
| $y_{ei} \leqq 0$, $i=16,17...37$ | Intermediate products (accounting) constraints |
| $y_{e38} \leqq 135.85$ | Resource allocation granted to Economics department by the college dean. |
| $y_{e39} \leqq 53.83$ | Same as above. |
| $y_{e40} \leqq 2.14$ | " " " |

[a] $y_{ei} = A_{ei}x_e$ for $i=1,2 \ldots 57$ where $A_{ei}$ is the ith row of $A_e$.

Table 62. (Continued).

| Commodity constraint[a] | Reason for constraint |
|---|---|
| $y_{e41} \leqq 0.73$ | Resource allocation granted to Economics department by the college dean. |
| $y_{e42} \leqq 26.4$ | Same as above. |
| $y_{e43} \leqq 511.33$ | " " " |
| $y_{e44} \leqq 7093.00$ | " " " |
| $y_{e45} \leqq 350{,}763.53$ | " " " |
| $y_{e46} \leqq 7200.$ | Resource constraints imposed by non-university suppliers of research funds. |
| $y_{e47} \leqq 80{,}000$ | Same as above. |
| $y_{e48} \leqq 125{,}000$ | " " " |
| $0 \leqq y_{e49} \leqq 6.0$ | Constraints due to students' decisions about graduate school attendance. |
| $0 \leqq y_{e50} \leqq 8.0$ | Same as above. |
| $0 \leqq y_{e51} \leqq 3.0$ | " " " |
| $0 \leqq y_{e52} \leqq 4.0$ | " " " |
| $y_{e53} = 5.0$ | Numbers of faculty positions authorized by university president. |
| $y_{e54} = 2.0$ | Same as above. |
| $y_{e55} = 6.5$ | " " " |
| $y_{e56} = 8.0$ | " " " |
| $y_{e57} = 3.0$ | " " " |

Table 63. Objective function weights associated with flexible targets, Economics department.

| Symbols used for flexible target levels[a] | Objective function weights |
|---|---|
| $y_{eo}$ $(= - x_{e22})^b$ | -4.8 |
| $y_{eoo}$ $(= - x_{e23})^b$ | -4.8 |
| $y_{eooo}$ $(= - x_{e20})^b$ | -2.4 |
| $y_{eoooo}$ $(= - x_{e21})^b$ | -2.4 |
| $y_{e12}$ $(= - x_{e1})$ | -2.0 |
| $y_{e13}$ $(= - x_{e2})$ | -2.0 |
| $y_{e14}$ $(= - x_{e3})$ | -4.0 |
| $y_{e15}$ $(= - x_{e4})$ | -4.0 |

[a]The symbols shown in the parenthesis were used for computing purposes since they allowed the use of only non-negative activity levels and the use of positive objective function weights. They are also easier to deal with in the discussions found in Chapter VII.

[b]For the commodities $e_o$ through $e_{oooo}$ no constraints were included in the model (see Table 62) but had constraints been included they would have had the form $y_{eo} = - x_{e22}$, etc.

Table 64. Solution to department E model after constraints are imposed.

| $x_e$ subscript | Solution value | $x_e$ subscript | Solution value |
|---|---|---|---|
| 1 | 1.25 | 42 | 9.97 |
| 2 | 0.60 | 43 | 0.0 |
| 3 | 5.73 | 44 | 0.0 |
| 4 | 9.76 | 45 | 0.0 |
| 5 | 1.25 | 46 | 0.0 |
| 6 | 0.0 | 47 | 10.43 |
| 7 | 0.16 | 48 | 13.03 |
| 8 | 0.34 | 49 | 0.0 |
| 9 | 0.0 | 50 | 0.0 |
| 10 | 0.0 | 51 | 0.0 |
| 11 | 1.63 | 52 | 6.87 |
| 12 | 0.0 | 53 | 12.87 |
| 13 | 0.02 | 54 | 0.0 |
| 14 | 0.0 | 55 | 0.0 |
| 15 | 0.0 | 56 | 0.0 |
| 16 | 2.15 | 57 | 0.0 |
| 17 | 1.48 | 58 | 7.51 |
| 18 | 0.0 | 59 | 0.12 |
| 19 | 100.0 | 60 | 1.64 |
| 20 | 5.92 | 61 | 4.90 |
| 21 | 13.21 | 62 | 9.37 |
| 22 | 9.06 | 63 | 8.54 |
| 23 | 8.29 | 64 | 26.57 |
| 24 | 26.0 | 65 | 0.0 |
| 25 | 0.0 | 66 | 3.32 |
| 26 | 0.0 | 67 | 1.68 |
| 27 | 13.2 | 68 | 0.0 |
| 28 | 0.0 | 69 | 0.0 |
| 29 | 0.0 | 70 | 2.00 |
| 30 | 0.0 | 71 | 0.0 |
| 31 | 36.0 | 72 | 3.71 |
| 32 | 0.0 | 73 | 2.76 |
| 33 | 0.0 | 74 | 0.03 |
| 34 | 20.0 | 75 | 6.36 |
| 35 | 0.0 | 76 | 1.64 |
| 36 | 20.0 | 77 | 0.0 |
| 37 | 2.88 | 78 | 0.0 |
| 38 | 12.82 | 79 | 0.0 |
| 39 | 15.89 | 80 | 0.0 |
| 40 | 11.81 | 81 | 1.83 |
| 41 | 8.86 | 82 | 1.17 |

Table 65. Solution to the dual of department E.

| Variable | Solution value[a] | | Variable | Solution value[a] | |
|---|---|---|---|---|---|
| $v_{e1}$ | 1.442 | (1.229) | $v_{e30}$ | 1.527 | (1.588) |
| $v_{e2}$ | .049 | (0.029) | $v_{e31}$ | 1.612 | (1.626) |
| $v_{e3}$ | .065 | (0.060) | $v_{e32}$ | 5.384 | (5.386) |
| $v_{e4}$ | .076 | (0.068) | $v_{e33}$ | 5.605 | (5.562) |
| $v_{e5}$ | .076 | (0.068) | $v_{e34}$ | 0.049 | (0.042) |
| $v_{e6}$ | .108 | (0.098) | $v_{e35}$ | 0.039 | (0.033) |
| $v_{e7}$ | .144 | (0.131) | $v_{e36}$ | 0.262 | (0.246) |
| $v_{e8}$ | .144 | (0.131) | $v_{e37}$ | 0.262 | (0.246) |
| $v_{e9}$ | 1.972 | (1.689) | $v_{e38}$ | 0.0 | (0.470) |
| $v_{e10}$ | 1.972 | (1.689) | $v_{e39}$ | 0.0 | (0.617) |
| $v_{e11}$ | 1.972 | (1.689) | $v_{e40}$ | 0.0 | (1.571) |
| $v_{e12}$ | 2.000 | (2.000) | $v_{e41}$ | 0.0 | (1.582) |
| $v_{e13}$ | 2.000 | (2.000) | $v_{e42}$ | 2.600 | (0.252) |
| $v_{e14}$ | 4.000 | (4.000) | $v_{e43}$ | 0.0 | (0.036) |
| $v_{e15}$ | 4.000 | (4.000) | $100v_{e44}$ | 0.446 | (0.466) |
| $v_{e16}$ | 1.972 | (1.689) | $1000v_{e45}$ | 1.209 | (1.020) |
| $v_{e17}$ | 1.972 | (1.689) | $1000v_{e46}$ | 0.461 | (0.446) |
| $v_{e18}$ | 1.972 | (1.689) | $1000v_{e47}$ | 0.261 | (0.255) |
| $v_{e19}$ | 1.972 | (1.689) | $1000v_{e48}$ | 0.230 | (0.230) |
| $v_{e20}$ | 0.695 | (0.592) | $v_{e49}$ | 0.0 | (0.0) |
| $v_{e21}$ | 0.200 | (0.171) | $v_{e50}$ | 0.036 | (0.033) |
| $v_{e22}$ | 4.735 | (4.120) | $v_{e51}$ | 0.799 | (0.768) |
| $v_{e23}$ | 0.895 | (0.709) | $v_{e52}$ | 0.691 | (0.716) |
| $v_{e24}$ | 0.791 | (0.648) | $v_{e53}$ | 0.788 | (0.827) |
| $v_{e25}$ | 0.725 | (0.728) | $v_{e54}$ | 2.333 | (1.936) |
| $v_{e26}$ | 0.668 | (0.669) | $v_{e55}$ | 0.525 | (0.598) |
| $v_{e27}$ | 2.192 | (2.170) | $v_{e56}$ | 2.155 | (1.980) |
| $v_{e28}$ | 1.510 | (1.518) | $v_{e57}$ | 1.433 | (1.364) |
| $v_{e29}$ | 1.404 | (1.433) | | | |

[a]The values shown in the parentheses are elements of the vector which solves the dual of 7.2 (version 2 of the college model).